Queueing Analysis of Multiserver Buffers with Geometric Service Times and Correlated Input Traffic

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Abstract

In this paper, we investigate the behavior of a discrete-time multiserver buffer system with infinite buffer size. Packets arrive in the system according to a two-state correlated arrival process. The service times of the packets are assumed to be independent and identically distributed according to a geometric distribution. We present an analytical technique, based on the use of generating functions, for the analysis of the system. Explicit expressions are obtained for the mean values, the variances and the tail distributions of the system contents and the packet delay. The influence of the various model parameters on the behavior of the system is shown by means of some numerical examples.

1 Introduction

Discrete-time buffer systems with different kinds of correlated packet arrival processes have been studied by many researchers during the last decades, given the bursty traffic characteristics in digital communication networks. In a discrete-time model, time is assumed to be slotted and the service or transmission of the packets is mostly assumed to be deterministic at the rate of one packet per slot, in accordance with the constant packet length used in e.g. ATM-based networks. Only a limited number of papers in literature consider more general service-time distributions, even though there are fundamentally theoretic as well as practical reasons to do this in view of the complicated and irregular service mechanisms in the internet. In ([3], [4], [13]), discrete-time buffer systems with (various types of) correlated arrivals and constant service times of arbitrary length ([3]), geometrically distributed service times ([4]) or a general service-time distribution ([13]) are analyzed, under the assumption that there is only one single server. Discrete-time multiserver systems with either constant service times of multiple slots ([5]) or geometric service times ([6], [7]) have been studied as well, but only for the case of an uncorrelated packet arrival process. The assumptions considered in [8] and [9] are uncorrelated arrivals, a single server and geometric ([8]) or general ([9]) service times.

In this paper, we investigate the behavior of a discrete-time buffer system with multiple servers, correlated arrivals and geometric service times. From the above survey, the paper can be seen as a generalization of [6] and [7] to the case of correlated arrivals or as an extension of [4] to the case of multiple servers. We present an analytical technique, based on the use of generating functions, for the analysis of the system behavior. The presented technique leads to explicit expressions for the mean values, the variances and the tail distributions of both the system contents and the packet delay, which can easily be evaluated numerically.

The outline of the paper is as follows. In Section 2, the system under study is described. In
Section 3, the probability generating function (pgf) of the system contents is derived, and from this pgf, the mean value, the variance and the tail distribution of the system contents are obtained. In Section 4, the characteristics of the packet delay are analyzed by means of a general relationship between system contents and delay. In Section 5, some numerical examples are given to illustrate the impact of the various model parameters on the system behavior. Finally, the paper is concluded in Section 6.

2 System Description

We consider a discrete-time buffer system with \( c \) \((c \geq 1)\) servers (output channels) and an infinite buffer size. The time axis is assumed to be divided into fixed-length time intervals, referred to as slots and chronologically labelled. Packets arrive at the input of the system and are queued in a buffer until they can be transmitted via one of the \( c \) output channels. The service (or transmission) of a packet is synchronized with respect to the slot boundaries, i.e., the service can start or end at slot boundaries only, and one packet can only get service from one of the servers. The packet service times are assumed to be independent and identically distributed (i.i.d.) random variables and geometrically distributed with parameter \( 1 - \mu \) \((0 \leq \mu \leq 1)\), i.e., with probability mass function (pmf)

\[
g(n) = \text{Prob}[\text{service of a packet takes } n \text{ slots}] = \mu(1 - \mu)^{n-1}, \quad n \geq 1,
\]

and corresponding pgf

\[
G(z) = \sum_{n=1}^{\infty} g(n) z^n = \frac{\mu z}{1 - (1 - \mu) z}.
\]

For the analysis of the packet delay (in Section 4), we moreover assume that packets are served according to a first-come-first-served (FCFS) queueing discipline.

The arrival process of packets is modelled as follows. The traffic source is of a bursty nature and alternates between two states, called state 0 and state 1. Transitions between the states are assumed to occur at slot boundaries. The numbers of consecutive slots during which the source state is 0 or 1 are called 0-times and 1-times, respectively. The 0-times and 1-times are assumed to be independent geometrically distributed random variables with parameters \( \alpha \) and \( \beta \) respectively, i.e.,

\[
\text{Prob}[0 \text{- times} = n \text{ slots}] = (1 - \alpha) \alpha^{n-1}, \quad n \geq 1;
\]
\[
\text{Prob}[1 \text{- times} = n \text{ slots}] = (1 - \beta) \beta^{n-1}, \quad n \geq 1.
\]

Note that this assumption implies a first-order Markovian correlation in the state of the source. The coefficient of correlation \( \gamma \) between the source states in two consecutive slots in the steady state can be derived from the above as \( \gamma = \alpha + \beta - 1 \). The case of an uncorrelated source state from slot to slot obviously corresponds to \( \alpha + \beta = 1 \) \((\gamma = 0)\). The number of packet arrivals to the buffer system during a slot has an arbitrary distribution which depends only on the state of the source during the slot. We denote the pmf’s of the numbers of arrivals during an arbitrary slot where the source state is 0 or 1 by \( e_0(n) \) or \( e_1(n) \), respectively, i.e.,

\[
e_0(n) = \text{Prob}[n \text{ arrivals in a slot where the source state is 0}];
\]
\[
e_1(n) = \text{Prob}[n \text{ arrivals in a slot where the source state is 1}],
\]

and the corresponding pgf’s by \( E_0(z) \) and \( E_1(z) \), respectively. It is also assumed that the service and arrival processes are mutually independent.
Finally, we assume a stable buffer system. That is, we assume the mean number of packet arrivals per slot to be strictly less than the mean number of packets that can be transmitted from the system per slot, or equivalently, the load
\[ \rho = \frac{p_0 E'_0(1) + p_1 E'_1(1)}{c \mu} \]  
(3) to be strictly less than 1. Here \( p_0 \) and \( p_1 \) denote the probabilities that the source is in state 0 or 1, respectively, during an arbitrary slot in the steady state:
\[ p_0 = \frac{1 - \beta}{2 - \alpha - \beta}, \quad p_1 = \frac{1 - \alpha}{2 - \alpha - \beta}, \]
and \( E'_0(1) \) and \( E'_1(1) \) are the first-order derivatives of \( E_0(z) \) and \( E_1(z) \) with respect to \( z \) at \( z = 1 \).

3 System Contents

This section deals with the analysis of the system contents (i.e., the total number of packets present in the buffer system, including the packets under transmission, if any). Specifically, in this section, we calculate the pgf of the system contents, and from this pgf, we derive several characteristics of the system contents, namely the mean, the variance and the tail distribution of the system contents.

3.1 The pgf of the system contents

We denote by \( v_k \) the system contents at the beginning of slot \( k \), by \( e_k \) the total number of packets arriving to the buffer during slot \( k \) and by \( s_k \) the state of the source during slot \( k \). In view of the modelling assumptions of Section 2, the evolution of the system contents from slot to slot is governed by the following system equation:
\[ v_{k+1} = v_k - t_k + e_k, \]  
(4)
where
\[ t_k = \sum_{j=1}^{(v_k,c)} t_{k,j}, \]  
(5)
and \((.,.)^{-}\) \(\overset{\triangle}{=} \min(.,.)\). In the above, the random variable \( t_k \) denotes the total number of departures at the end of slot \( k \), i.e., the total number of packets whose service is completed at the end of slot \( k \). The random variables \( t_{k,j} \) correspond to the number of departures at the end of slot \( k \) via the \( j \)-th output channel when there is a packet under transmission over this output channel during slot \( k \). Since the packet service times are geometrically distributed, \( t_{k,j} \) is a Bernoulli distributed random variable with parameter \( \mu \), i.e., \( \text{Prob}[t_{k,j} = 1] = \mu, \text{Prob}[t_{k,j} = 0] = 1 - \mu \). Consequently, the conditional pgf
\[ T_i(z) \overset{\triangle}{=} E[z^{s_k} | (v_k,c)^- = i], \quad i = 0, 1, ..., c, \]
where \( E[\cdot] \) denotes the expected value of the quantity between brackets, is easily seen to be given by
\[ T_i(z) = (1 - \mu + \mu z)^i. \]  
(6)
From the arrival process description of Section 2, it moreover follows that \( \{s_k\} \) is a homogeneous two-state Markov chain and the distribution of \( e_k \) depends on the value of \( s_k \) only. More specifically, it is not difficult to show that the joint pgf of the random variables \( (s_{k+1}, e_k) \) can be written
in terms of the pgf of the random variable $s_k$:

$$E[x^{s_{k+1}} z^{v_k}] = S_0(x,z) E \left[ \left( \frac{S_1(x,z)}{S_0(x,z)} \right)^{s_k} \right],$$

(7)

where

$$S_0(x,z) = [\alpha + (1 - \alpha) x] E_0(z);$$

$$S_1(x,z) = [1 - \beta + \beta x] E_1(z).$$

Together with equations (4)-(6), we may then conclude that the pair $(s_k, v_k)$ constitutes a Markovian state description of the system at the beginning of slot $k$.

Let us now define $P_k(x,z)$ as the joint pgf of the state vector $(s_k, v_k)$, i.e.,

$$P_k(x,z) \triangleq E[x^{s_k} z^{v_k}].$$

(8)

Using equations (4)-(8) and some standard $z$-transform techniques, we can then express $P_{k+1}(x,z)$ in terms of the $P_k$-function. Since we assume a stable system, the pgf’s $P_k(x,z)$ and $P_{k+1}(x,z)$ converge both to a common steady-state value

$$P(x,z) \triangleq \lim_{k \to \infty} P_k(x,z).$$

By taking the limit $k \to \infty$ in the relationship between the $P_k$-function and the $P_{k+1}$-function, we then find the following functional equation for $P(x,z)$:

$$P(x,z) = S_0(x,z) \left\{ P \left( \frac{S_1(x,z)}{S_0(x,z)} z \right) T_c \left( \frac{1}{z} \right) \right.$$  

$$- \sum_{j=0}^{c-1} \sum_{i=0}^{c-1} \left( \frac{S_1(x,z)}{S_0(x,z)} \right)^j \Phi_i(z) \text{Prob}[v = i, s = j] \left\} ,$$

(9)

where $(s, v)$ denotes the steady-state version of $(s_k, v_k)$, and

$$\Phi_i(z) = \left[ T_c \left( \frac{1}{z} \right) - T_i \left( \frac{1}{z} \right) \right] z^i.$$

Next, let us introduce the following partial pgf’s:

$$P_j(z) \triangleq \lim_{k \to \infty} \sum_{n=0}^{\infty} \text{Prob} \left[ s_k = j, v_k = n \right] z^n, \quad j = 0, 1.$$  

(10)

Then the function $P(x,z)$ can be expressed as

$$P(x,z) = P_0(z) + xP_1(z).$$

(11)

Substitution of (11) in the functional equation (9) and identification of the coefficients of equal powers of $x$ on both sides of the resulting equation then yields a set of two linear equations for $P_0(z)$ and $P_1(z)$, from which these partial pgf’s can be calculated explicitly. The pgf $V(z)$ of the system contents $v$ at the beginning of an arbitrary slot in the steady state is then obtained as

$$V(z) = P(1,z) = P_0(z) + P_1(z)$$

$$= \sum_{i=0}^{c-1} \left[ \gamma T_c \left( \frac{1}{z} \right) E_0(z) E_1(z) v(i) - E_a(i,z) \right] \Phi_i(z)$$

$$\frac{\gamma E_0(z) E_1(z) T_c \left( \frac{1}{z} \right)^2 - T_c \left( \frac{1}{z} \right) E_2(z) + 1}{\gamma E_0(z) E_1(z) T_c \left( \frac{1}{z} \right)^2 - T_c \left( \frac{1}{z} \right) E_2(z) + 1} ,$$

(12)
where
\[
v(i) = \text{Prob}[v = i] = \text{Prob}[v = i, s = 0] + \text{Prob}[v = i, s = 1],
\]
\[
E_a(i, z) = \text{Prob}[v = i, s = 0]E_0(z) + \text{Prob}[v = i, s = 1]E_1(z),
\]
\[
i = 0, 1, \ldots, c - 1;
\]
\[
E_2(z) = \alpha E_0(z) + \beta E_1(z).
\]

It now remains for us to determine the 2c unknown constants \(\text{Prob}[v = i, s = 0]\) and \(\text{Prob}[v = i, s = 1]\) \((i = 0, 1, \ldots, c - 1)\) in the expression of \(V(z)\). These can be obtained by invoking the analyticity of the pgf \(V(z)\) inside the unit disk \((z : |z| < 1)\) of the complex \(z\)-plane and the normalization condition \(V(1) = 1\). Specifically, by means of Rouché’s theorem ([1]), it can be shown that the denominator of the right-hand side of (12) has exactly \(2c - 1\) roots inside the unit disk. We denote these roots by \(z_r, r = 1, 2, \ldots, 2c - 1\). Since \(V(z)\) is analytic for \(|z| < 1\), the numerator of the right-hand side of (12) must also be zero at these points. Thus, we have

\[
\sum_{i=0}^{c-1} \left[ \gamma T_c \left( \frac{1}{z_r} \right) E_0(z_r) E_1(z_r) v(i) - E_a(i, z_r) \right] \Phi_i(z_r) = 0, \quad r = 1, 2, \ldots, 2c - 1. \tag{13}
\]

From the normalization condition \(V(1) = 1\) and equation (12), we find that

\[
\mu \sum_{i=0}^{c-1} (c - i) v(i) = c\mu - E'(1), \tag{14}
\]

where \(E'(1)\) denotes the mean number of packet arrivals during an arbitrary slot, i.e., \(E'(1) = p_0 E'_0(1) + p_1 E'_1(1)\). From the set of equations (13) and (14), the constants \(\text{Prob}[v = i, s = 0]\) and \(\text{Prob}[v = i, s = 1]\) \((i = 0, 1, \ldots, c - 1)\) can be calculated, and hence \(V(z)\) can be determined explicitly. In the rest of Section 3, we will use the pgf \(V(z)\) to derive some important characteristics of the system contents at the start of an arbitrary slot in the steady state.

### 3.2 Mean value and variance of the system contents

In order to derive the mean system contents, we evaluate the first-order derivative of the pgf \(V(z)\) with respect to \(z\) at \(z = 1\). From equation (12) and after some mathematical manipulations, we get

\[
E[v] = V'(1)
\]

\[
= \frac{E''(1) - (\mu - 2)E'(1)}{2(\mu - E'(1))} + \frac{c\mu(\gamma + 1)E'(1) - 2\gamma [p_0 E'_0(1)^2 + p_1 E'_1(1)^2]}{2(\gamma - 1)(c\mu - E'(1))}
\]

\[
+ \frac{\mu}{c\mu - E'(1)} \sum_{i=0}^{c-1} (c - i) \left[ 1 - \frac{\mu}{2} iv(i) - \frac{E_a(i, 1)}{\gamma - 1} \right],
\]

where \(E'_a(i, 1)\) is the first-order derivative of \(E_a(i, z)\) with respect to \(z\) at \(z = 1\) and \(E''(1) = p_0 E''_0(1) + p_1 E''_1(1)\). In a similar way, higher-order moments of the system contents can be derived by calculating the appropriate higher-order derivatives of \(V(z)\) at \(z = 1\). For instance, the variance of the system contents can be obtained from

\[
Var[v] = V'''(1) + V'(1)^2 - V'(1)^2,
\]

where \(V'''(1)\) can be calculated by taking the second-order derivative of equation (12) with respect to \(z\) at \(z = 1\).
3.2.1 Tail probabilities of the system contents

A characteristic of considerable interest is the probability that the system contents exceeds a certain threshold $N$. This quantity is often used to approximate the packet loss probability, i.e., the fraction of the arriving packets that are lost upon arrival because of buffer overflow, in a finite buffer of size $N$ (see e.g. [14]). As argued in ([2], [10], [11]), the system-contents distribution exhibits a geometric tail behavior. That is, for sufficiently large values of $N$, the tail distribution of the system contents can be approximated as

$$\text{Prob}[v > N] \approx -C_v \frac{z_v^{-N-1}}{z_v - 1}. \quad (17)$$

In the above expression, $z_v$ is the pole of $V(z)$ with the smallest modulus (outside the unit disk), and the constant $C_v$ is the residue of $V(z)$ at $z = z_v$. As shown in [11], the dominant pole $z_v$ must necessarily be real and positive in order to ensure that the tail distribution is nonnegative anywhere. From equation (12), it follows that $z_v$ is a real positive zero of the denominator of $V(z)$.

The residue $C_v$ can be calculated from (12) as

$$C_v = \frac{\sum_{i=0}^{c-1} \Phi_i(z_v) [X(z_v)v(i) - E_0(i, z_v)]}{\gamma T_c(1/z_v)^2 E_0(z_v) - T_c(1/z_v)E_2(z_v) + F(z_v)}, \quad (18)$$

where

$$X(z_v) = E_2(z_v) - 1/T_c(1/z_v);$$

$$E_0(z_v) = E_0(z_v)E_1'(z_v) + E_1(z_v)E_0'(z_v);$$

$$F(z_v) = \frac{\gamma \mu}{z_v^2 T_1(1/z_v)} \left[ 2 - T_c(1/z_v)E_2(z_v) \right].$$

4 Packet Delay

The delay of a packet is defined as the total number of slots between the end of the slot during which the packet arrived in the buffer system and the end of the slot where the packet’s transmission finishes and the packet leaves the system. In this section, we analyze the characteristics of the packet delay by means of a general relationship between system contents and packet delay established in [15].

4.1 The pgf of the delay

Let us consider an arbitrary packet, denoted by $P$ and called the tagged packet. Upon arrival, the packet $P$ will find a number of other packets in the system. Because of the FCFS queueing discipline, just after the end of the arrival slot of $P$, all packets that arrived in the previous slots, but have not been taken into service yet, and all packets that arrived in the same slot as $P$, but before $P$, are waiting in the buffer in front of the tagged packet. Whenever there are output channels available for transmission at the beginning of a slot, the packet at the head of the queue is selected for service, until eventually the tagged packet $P$ itself is served and leaves the system. We denote by $q$ the number of packets staying in the buffer system at the arrival instant of $P$, except the tagged packet itself and the ones that leave the system at the end of the arrival slot of $P$, if such packets exist. Also let $Q(z)$ denote the pgf of $q$. In [15], the following relationship was established between $Q(z)$ and the steady-state pgf $V(z)$ of the system contents at the beginning of an arbitrary slot, for discrete-time multiserver buffer systems with geometric service times and general, possibly correlated, arrivals:

$$Q(z) = \frac{\Phi_0(z)V(z) - \sum_{i=0}^{c-1} \Phi_i(z)v(i)}{E'(1)(1 - z)}. \quad (19)$$
Moreover, it was shown in [15] that for the same class of buffer systems, the pgf $D(z)$ of the delay of an arbitrary packet can be expressed in terms of the $Q$-function as

$$
D(z) = \frac{\mu(z - 1)}{1 - (1 - \mu)z} \sum_{p=0}^{c-1} x_p^{c-1} T_p(1-x_p) Q\left(1 - \frac{x_p}{x_p(1-x_p)}\right),
$$

(20)

where the $x_p$'s ($p = 0, 1, ..., c - 1$) are the $c$ solutions for $x$ in terms of $z$ of the equation

$$
1 - z T_c(x) = 0.
$$

(21)

As shown in [15], the relationships (19) and (20) are valid for any discrete-time multiserver system with geometric service times, regardless of the exact nature of the arrival process, and therefore, they can also be applied here to derive the pgf $D(z)$ of the packet delay for the system with a two-state correlated traffic source considered in this paper. By combining equations (12), (19) and (20), we then get

$$
D(z) = \frac{(1 - z)^2}{cE'(1)(1-x_p \sum_{p=0}^{c-1} x_p^{c-1} T_p(1-x_p))} \sum_{i=0}^{c-1} X_{p,i}(z)
\times \left[ \frac{v(i)}{1 - z} - \frac{\gamma E_0(1/x_p) E_1(1/x_p) v(i) - z E_a(i, 1/x_p)}{z^2 - z E_2(1/x_p) + \gamma E_0(1/x_p) E_1(1/x_p)} \right],
$$

(22)

where $X_{p,i}(z) = |T_i(x_p) z - 1| (1/x_p)^i$.

### 4.2 Mean value and variance of the delay

In a similar way as explained for the system contents in Subsection 3.2, the moments of the packet delay can be found by calculating the appropriate derivatives of $D(z)$ at $z = 1$ from equation (22). As a result, we find the mean packet delay as

$$
E[d] = E[v] \frac{E'(1)}{E'(1)}.
$$

(23)

So, as expected, Little’s law ([12]) is satisfied. Calculation of the second-order derivative of $D(z)$ at $z = 1$ yields an expression for the variance of the packet delay (delay jitter) through the relation

$$
Var[d] = D''(1) + D'(1) - D'(1)^2.
$$

(24)

### 4.3 Tail probabilities of the delay

The tail distribution of the packet delay, i.e., the probability that the delay exceeds a given threshold $T$, for a sufficiently large value of $T$, can be expressed as

$$
Prob[d > T] \approx -C_d \frac{z^T - 1}{z_T - 1},
$$

(25)

when $D(z)$ has a dominant pole $z_d$ with multiplicity 1, and where $C_d$ is the residue of $D(z)$ at $z = z_d$, or

$$
Prob[d > T] \approx -\frac{z^T - 1}{z_T - 1} \left[ C_{2d} - \left(2 + T + \frac{1}{z_d - 1}\right) C_{1d} \right],
$$

(26)

when $D(z)$ has a dominant pole $z_d$ with multiplicity 2, and where $C_{1d}$ and $C_{2d}$ are determined by

$$
C_{1d} = \lim_{z \to z_d} (z - z_d)^2 D(z), \quad C_{2d} = \lim_{z \to z_d} \frac{d}{dz} [(z - z_d)^2 D(z)].
$$

(27)
In a similar way as explained in [7], it can be argued that the dominant pole $z_d$ of $D(z)$ either equals $1/(1 - \mu)$ or $1/T_c(1/z_v)$, depending on which value has the smallest modulus. Consequently, we get the following three cases:

**CASE 1.** When $1/T_c(1/z_v) < 1/(1 - \mu)$, we get a dominant pole $z_d$ of multiplicity 1, given by

$$z_d = \frac{1}{T_c(1/z_v)},$$

and the tail distribution of the delay has the geometric form (25), where

$$C_d = C_v \frac{G(z_d)(z_d - 1)^2}{E'(1) z_v^c(z_v - 1)^2}.$$

Note that this case always occurs for $c = 1$ because of the monotonically increasing character of $T_c(z)$ along the positive real axis, i.e.,

$$\frac{1}{T_1(1/z_v)} < \frac{1}{T_1(0)} = \frac{1}{1 - \mu}.$$

**CASE 2.** When $1/T_c(1/z_v) > 1/(1 - \mu)$, the dominant pole $z_d$ also has a multiplicity 1 and is equal to

$$z_d = \frac{1}{1 - \mu}.$$

In this case, we also have a tail distribution of the form (25), where $C_d$ is given by

$$C_d = -\frac{\mu^2}{(1 - \mu)^2} \sum_{p=0}^{c-1} \frac{x_p^{c-1}}{T_c(x_p)(1 - x_p)} Q\left(\frac{1}{x_p}\right),$$

and the $x_p$'s $(p = 0, 1, \ldots, c - 1)$ are the $c$ solutions for $x$ of the equation

$$T_c(x) = 1 - \mu.$$

**CASE 3.** When $1/T_c(1/z_v) = 1/(1 - \mu)$, there is a dominant pole $z_d$ of multiplicity 2, given by

$$z_d = \frac{1}{1 - \mu} = \frac{1}{T_c(1/z_v)}.$$

and the tail distribution of the delay has the form (26), where $C_{1d}$ and $C_{2d}$ are obtained from (20) and (27) as

$$C_{1d} = -\frac{\mu C_v}{(1 - \mu)^2 E'(1)} \frac{(z_d - 1)^2}{(z_v - 1)^2 z_v};$$

$$C_{2d} = (1 - \mu) C_{1d} T_1(1/z_v) z_v \left\{ \frac{z_d \sum_{i=0}^{c-1} \Phi_i(z_v) z_v(i)}{c \mu C_v (z_d - 1)} - \frac{1 + z_v}{c \mu (1 - z_v)} + \frac{z_v}{T_1(z_v)} \left[ \frac{c(c+1)\mu^2}{2} + \frac{z_d + 1}{z_d - 1} \right] + \frac{(c-1)(z_v - \mu z_v + 2\mu)}{c \mu z_v T_1(1/z_v)} + \frac{2V_n'(z_v) - C_v V_d'(z_v)}{2c \mu C_v V_d'(z_v)} \right\} - \frac{\mu(z_d - 1)}{1 - \mu} \sum_{p} \frac{x_p^{c-1} Q(1/x_p)}{T_c(x_p)(1 - x_p)}.$$

Here the $x_p$’s in the last term of $C_{2d}$ are the solutions for $x$ of $T_c(x) = 1/z_d$, except the solution
Bernoulli distribution with the arrival rate $\lambda$ packet arrivals when the source is in state 1. In the second set, packet arrivals are governed by a geometric distribution during state 0 and there are no arrivals during states 0 and 1 are governed by the sets of distributions shown in Table 1. In the first set, packet arrivals are governed by a Bernoulli distribution with the arrival rate $\lambda$ during state 0 and a geometric distribution with the arrival rate $2\lambda$ during state 1. In the third set, the arrival distributions are of the same type as for the second set, but with the same arrival rate $\lambda$ during state 0 and state 1.

In Figure 1, we illustrate the influence of the correlation in the source state on the system behavior. Specifically, we have plotted the mean system contents (for set 2 of Table 1) versus the load $\rho$, for $\mu = 0.8$, $c = 4$, $\alpha = \beta$ and various values of the state correlation coefficient $\gamma$, namely $\gamma = -0.6$, 0, 0.6, 0.8. The figure clearly shows that for a given $\rho$, the mean system contents increases as $\gamma$ increases. Especially, for higher loads $\rho$, the system contents may be heavily underestimated when the (positive) correlation between the source states in two consecutive slots is not taken into account.

In the Figures 2-4, we assume $\alpha = 0.7$, $\beta = 0.8$ and $\mu = 0.8$. The correlation coefficient $\gamma$ between the source states in consecutive slots then equals 0.5. In Figure 2, the tail distribution of the system contents has been plotted for $\rho = 0.75$, $c = 4$, and for the three sets of arrival

<table>
<thead>
<tr>
<th>Table 1: The three sets of arrival distributions.</th>
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<tbody>
<tr>
<td>Set</td>
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<tr>
<td>-----</td>
</tr>
<tr>
<td>$E_0(z)$</td>
</tr>
<tr>
<td>$E_1(z)$</td>
</tr>
</tbody>
</table>

where $1/x_p = z_v$, and

$$
V'_d(z_v) = \frac{\gamma E_s(z_v)}{z_d^2} - \frac{E'_2(z_v)}{z_d} + F(z_v);
$$

$$
V''_d(z_v) = \gamma \left[ E_1(z_v)E''_0(z_v) + 2E'_0(z_v)E'_1(z_v) + E_0(z_v)E''_1(z_v) \right]
- \frac{2c\mu}{z_v^2} \left[ \frac{2\gamma E_s(z_v) - z_d E'_2(z_v)}{T_1(1/z_v)} \right] + \frac{c^2\mu^2 E_2(z_v)}{z_d^2 T_1(1/z_v)^2}
- \left[ \frac{2}{z_v} - \mu(1 - 2c) \right] \frac{E''_2(z_v)}{z_d};
$$

$$
V''_a(z_v) = \sum_{i=0}^{v-1} \left\{ \Phi_i(z_v) \left[ \left( E_s(z_v) - \frac{c\mu E_0(z_v) E_1(z_v)}{T_1(1/z_v) z_v} \right) \frac{\gamma v(i)}{z_d} - E_a(i, z_v) \right]
- \frac{i}{z_v} \left( \mu z_v^i T_i(1/z_v) \Phi_i(z_v) \right) - \frac{c\mu z_v^{i-2}}{z_d T_1(1/z_v)} \right\}
\times \left[ E_a(i, z_v) - \frac{E_2(z_v) - z_d v(i)}{z_d} \right].
$$

5 Numerical Results

We now present a number of numerical examples in order to illustrate the influence of various parameters of the model, such as the degree of correlation in the arrival process or the number of output channels, on the system performance. Throughout this section, we assume that the packet arrivals during states 0 and 1 are governed by the sets of distributions shown in Table 1.
Figure 1: Mean system contents versus load $\rho$.

Figure 2: Tail distribution of the system contents, $\text{Prob}[v > N]$, versus $N$. 
Figure 3: Mean packet delay versus load $\rho$.

Figure 4: Variance of the packet delay versus load $\rho$. 
distributions. The first set gives the highest value for the packet loss and the third set gives the smallest value. This can be understood intuitively from the variance of the number of arrivals in a slot. The greater the variance of the number of arrivals and, hence, the more fluctuation of the arrival process, the higher the buffer contents, and the more packets will get lost for a given amount of buffer space. In our case, the variance of the number of arrivals per slot decreases in the order of set 1, set 2 and set 3. The required buffer size $N$ to satisfy a given loss bound can also be found from Figure 2.

In Figure 3, the mean packet delay is plotted versus the load $\rho$ for set 1 in Table 1, and for $c = 1, 4, 8$. For a given number of output channels and a given set of arrival distributions, we observe that the mean packet delay increases with increasing values of $\rho$. For a given $\rho$, the mean delay for the system with more output channels is smaller than the delay for systems with less output channels, as expected intuitively.

In Figure 4, the variance of the packet delay is shown versus $\rho$, for $c = 4$, and for the three sets of arrival distributions. For a given number of output channels and a given $\rho$, the value of the variance of the packet delay for the three sets of arrivals decreases in the order of set 1, set 2 and set 3. This can again be explained based on the magnitude of the variance of the number of arrivals per slot, which also decreases in the order of set 1, set 2 and set 3.

6 Conclusions

In this paper, we have studied the behavior of a discrete-time buffer system with multiple servers and geometrically distributed service times. Packets enter the system according to a correlated arrival process. Specifically, a two-state traffic source with a first-order Markovian correlation in the state of the source is considered. We have presented an analytical technique based on generating functions for the analysis of the system. As a result, closed-form expressions have been derived for the mean values, the variances and the tail distributions of the system contents and the packet delay. The obtained results are easy to evaluate numerically. Some numerical results have been presented to illustrate the analysis.

References


