HOL priority in an ATM output queuing switch

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Abstract

In this paper, we consider a discrete-time queuing system with head-of-line (HOL) priority. In a first part of this paper, we will give some general results on the effect of priority scheduling in a queue. In particular, we will derive expressions for the Probability Generating Function of the system contents and the cell delay. We will then apply these expressions to the important case of an NxN ATM output queuing switch. Some performance measures (such as mean and variance) will be derived, and used to illustrate the impact and significance of priority scheduling in an ATM switch by some numerical examples.

1 Introduction

In recent years, there has been much interest in ATM. Especially its well-defined QoS guarantee makes it extremely suitable for multimedia applications. Different types of traffic need different QoS standards. For real-time applications, it is important that mean delay and delay-jitter are not too large, while for non real-time applications, the Cell Loss Ratio (CLR) is the restrictive quantity.

In general, one can distinguish two priority classes, which will be referred to as Time Priority and Space Priority. Time priority scheduling tries to reduce the delay of delay-sensitive traffic (such as voice). This is done by using a more sophisticated type of scheduling than the simple FIFO scheduling. Priority is given to delay-sensitive traffic over delay-insensitive traffic. Several types of Time priority (or cell scheduling) schemes (such as Weighted-Round-Robin (WRR), Weighted-Fair-Queueing (WFQ)) have been proposed and analyzed for ATM applications, each with their own specific algorithmic and computational complexity (see e.g. [8] and the references therein). On the other hand, Space Priority schemes attempt to reduce the cell loss of loss-sensitive traffic (such as data). Again, various types of Space Priority (or cell discarding) strategies for ATM (such as Push-Out Buffer (POB), Partial Buffer Sharing (PBS)) have been presented in the literature (see e.g. [15]). An overview of both types of priority can be found in [1].

In this paper, we will focus on the effect of HOL time priority scheduling. We assume that time-sensitive traffic has absolute priority over time-insensitive traffic, i.e., when a server becomes idle, a cell of time-sensitive traffic, when available, will always be scheduled next. This is the most drastic type of time priority scheduling, but also the easiest one to implement. In literature, there have been a number of contributions with respect to this priority scheme. In [7,9-14], HOL priority queues have been analyzed with a wide variety of arrival and service time distributions.
The contribution of this paper is two-fold. First, we want to show that an analysis based on generating functions is extremely suitable for modelling ATM buffers with a priority scheduling. From these generating functions, we can then easily calculate expressions for some interesting performance measures, such as mean value, variance and the correlation coefficient amongst classes of the buffer contents and cell delay. These closed-form expressions require virtually no computational effort at all, and are well-suited for evaluating the impact of the various system parameters on the overall performance. Secondly, we will show that our approach can be applied to the case of an ATM output-queueing switch with HOL priority scheduling. There have been a number of contributions with respect to switches with output queuing and a single traffic type, such as [4-6]. The main difference with the articles involved with HOL priority queues listed above is that, for the case of a multiclass output-queueing switch, the arrival processes of the different types of cells are not mutually independent. Therefore the different classes can not be analyzed separately (i.e., as a model with server interruptions for low priority cells as demonstrated in section 6), which complicates the analysis.

The outline is as follows. First, we are going to consider a single queue with a general arrival distribution. In the following section, we will introduce the mathematical model. In section 3 and 4 we will analyze the steady-state system contents and cell delay. In section 5, we calculate the moments of the system contents and cell delay. In section 6, we discuss the results derived in the former sections while we apply these results to an output queueing switch with Bernoulli arrivals, and discuss the impact of priority in section 7. Some conclusions are formulated in section 8.

2 Mathematical model

We consider a discrete-time single-server queueing system with infinite buffer space. Time is assumed to be slotted, where 1 slot equals the transmission time of a cell. There are 2 types of traffic arriving in the system, namely cells of class 1 and cells of class 2. We denote the number of arrivals of class $j$ during slot $k$ by $a_{j,k}$ ($j = 1, 2$). Both types of cell arrivals are assumed to be i.i.d. from slot-to-slot and are characterized by the joint probability mass function $a(m,n)$,

$$a(m,n) \triangleq Pr[a_{1,k} = m, a_{2,k} = n],$$

and joint probability generating function (pgf) $A(z_1, z_2)$,

$$A(z_1, z_2) \triangleq E[z_1^{a_{1,k}} z_2^{a_{2,k}}].$$

Notice that the number of cell arrivals from different classes (within a slot) can be correlated. Further, we denote the total number of arriving cells during slot $k$ by $a_T,k \triangleq a_{1,k} + a_{2,k}$ and its pgf is defined as $A_T(z) \triangleq E[z^{a_{T,k}}] = A(z,1)$. In the same way, we define the marginal pgf's of the arrivals from class 1 and class 2 during a slot by $A_1(z) \triangleq E[z^{a_{1,k}}] = A(z,1)$ and $A_2(z) \triangleq E[z^{a_{2,k}}] = A(1,z)$ respectively. We furthermore denote the arrival rate of class $j$ ($j = 1, 2$) by $\lambda_j = A_j'(1)$ and the total arrival rate by $\lambda_T = A_T'(1) = A_1'(1) + A_2'(1)$. We assume a stable system, i.e., $\lambda_T < 1$.

The system has one server that provides the transmission of cells, at a rate of 1 cell per slot. Class 1 cells are assumed to have priority over class 2 cells, and within one class the service discipline is FCFS. Due to the priority scheduling mechanism, it is as if class 1 cells are stored in front of class 2 cells in the queue. So, if there are any class 1 cells in the queue at the beginning of a slot, the one with the longest waiting time will be served next. If, on the other hand, no class 1 cells are present in the queue at the beginning of a particular slot, the class 2 cell with the longest waiting time, if any, will be served.
3 System contents

In this section, we concentrate on the effect of HOL priority scheduling on the steady-state distribution of system contents. We denote the system contents of class \( j \) at the beginning of slot \( k \) by \( u_{j,k} \) \((j = 1, 2)\) and the total system contents at the beginning of slot \( k \) by \( u_{T,k} \). Furthermore, we denote the joint pdf of \( u_{1,k} \) and \( u_{2,k} \) by \( U_k(z_1, z_2) \), i.e.,

\[
U_k(z_1, z_2) \triangleq E[z_1^{u_{1,k}} z_2^{u_{2,k}}].
\]

The system contents of both types of cells is characterized by the following system equations:

\[
\begin{align*}
  u_{1,k+1} &= [u_{1,k} - 1]^+ + a_{1,k} ; \\
  u_{2,k+1} &= \begin{cases} [u_{2,k} - 1]^+ + a_{2,k} & \text{if } u_{1,k} = 0 \\
  u_{2,k} + a_{2,k} & \text{if } u_{1,k} > 0 ,
\end{cases}
\end{align*}
\]

where \([\cdot]^+\) denotes the maximum of the argument and 0. The first equation follows from the observation that class 1 cells are not influenced by class 2 cells. A class 2 cell on the other hand can only be served, if there are no class 1 cells in the system. This leads to the second equation. Using these system equations, we can form the following relation between \( U_{k+1}(z_1, z_2) \) and \( U_k(z_1, z_2) \)

\[
U_{k+1}(z_1, z_2) = A(z_1, z_2) z_2 U_k(z_1, z_2) + (z_1 - z_2) U_k(0, z_2) + z_1(z_2 - 1) U_k(0, 0) \quad \frac{z_1 z_2}{z_2(z_1 - A(z_1, z_2))}.
\]

(1)

Since we are interested in the steady-state distribution of the system contents, we define \( U(z_1, z_2) \) as

\[
U(z_1, z_2) \triangleq \lim_{k \to \infty} U_k(z_1, z_2).
\]

Applying this limit in equation (1), we find the following expression for \( U(z_1, z_2) \),

\[
U(z_1, z_2) = A(z_1, z_2) \frac{(z_1 - z_2) U(0, z_2) + z_1(z_2 - 1) U(0, 0)}{z_2(z_1 - A(z_1, z_2))}.
\]

(2)

There are two quantities yet to be determined in the right hand side of equation (2), namely the function \( U(0, z_2) \) and the constant \( U(0, 0) \). Applying Rouche's theorem, it can be proven that for a given value of \( z_2 \) (\(|z_2| < 1\) ), the equation \( z_1 = A(z_1, z_2) \) has one solution in the unit circle for \( z_1 \), which will be denoted by \( Y(z_2) \) in the remainder, and which is implicitly defined by \( Y(z) \triangleq A(Y(z), z) \). Since a generating function is finite in the unit circle, \( Y(z_2) \) must be a zero of the numerator. We thus find

\[
U(0, z_2) = U(0, 0) \frac{Y(z_2)(z_2 - 1)}{z_2 - Y(z_2)}.
\]

Substituting this result in equation (2) yields

\[
U(z_1, z_2) = U(0, 0) \frac{A(z_1, z_2)(z_2 - 1) z_1 - Y(z_2)}{z_1 - A(z_1, z_2) z_2 - Y(z_2)}.
\]

(3)
U(0, 0) can be found by applying the normalization condition U(1, 1) = 1. Using de l’Hopital’s rule gives the expected result for the probability of having an empty system: U(0, 0) = 1 − λT. From equation (3), we easily obtain an expression for the pgf UT(z) describing the total system contents

\[ UT(z) = U(z, z) = U(0, 0) \frac{A_T(z)(z - 1)}{z - A_T(z)}. \]  

(4)

We can also calculate the pgf Uj(z) (j = 1, 2) of the system contents of class j, namely

\[ U_1(z) = U(z, 1) = U(0, 1) \frac{A_1(z)(z - 1)}{z - A_1(z)}, \]

(5)

\[ U_2(z) = U(1, z) = U(0, 0) \frac{1 - Y(z)}{1 - A_2(z)} \frac{A_2(z)(z - 1)}{z - Y(z)}. \]

(6)

We will discuss these results in section 6.

4 Cell Delay

The cell delay is defined as the total amount of time that a cell spends in the system, i.e., the number of slots between the end of the cell’s arrival slot and the end of its departure slot. We can analyze the cell delay of class 1 cells as if they are the only cells in the system. This is e.g. done in [3] and the pgf of the cell delay of class 1 cells is given by

\[ D_1(z) = \frac{1 - \lambda_1}{\lambda_1} \frac{z(A_1(z) - 1)}{z - A_1(z)}. \]

(7)

The analysis of the cell delay of a class 2 cell is more complicated. We tag an arbitrary class 2 cell. The amount of time it spends in the system equals

\[ d_2 = \sum_{j=1}^{[uT,k-1]+f_2} v_j^0 + 1, \]

(8)

where slot k is assumed to be the arrival slot of the tagged cell, f_2 is defined as the total number of cells that arrive during the arrival slot of the tagged cell, but which are served before it, and v_j^0 represents the number of slots it takes for the tagged cell to move one position ahead in the queue, e.g., from position j to position j−1 (see Figure 1). In case of FIFO scheduling, v_j^0 would equal 1. For HOL priority scheduling, this is not necessarily the case, since new class 1 cells can arrive while the tagged cell is waiting in the queue and these class 1 cells have to be served before the tagged cell. More specific, assume that the tagged cell is stored in the j-th position in the queue at the beginning of the l-th slot (0 < j < [uT,k-1]+f_2) and a class 2 cell is served during slot l. If no class 1 cells arrive during slot l, v_j^0 will equal 1. If a_{1,l} (> 0) class 1 cells arrive during this slot on the other hand, the tagged cell will move back to position j + a_{1,l} − 1 at the beginning of slot l+1 in the queue, since these class 1 cells have to be served before all class 2 cells, and thus before the tagged one (Figure 1). If we then define v_j^i (j ≤ i ≤ j + a_{1,l} − 1) as the number of slots it takes the tagged cell to go from position i to position i−1, it is clear that v_j^0 can be calculated as follows,
\[ v_j^0 = \sum_{i=1}^{a_j} v_j^1 + 1. \]  \hspace{1cm} (9)

Now, one can easily see that all \( v_j^0 \) and \( v_j^1 \) form a set of mutually independent random variables. From a stochastic point-of-view, these are i.i.d. variables and, as a result, are characterized by the same pgf \( V(z) \). From equation (9), it can be seen that \( V(z) \) satisfies

\[ V(z) = zA_1(V(z)). \]  \hspace{1cm} (10)

Furthermore, \( f_2 \) is the sum of all the class 1 cells that arrive during the same slot as the tagged one, and of the class 2 cells that have arrived before it during its arrival slot. The pgf of \( f_2 \) can be calculated taking into account that a cell is more likely to arrive in a larger bulk (e.g. [3]), yielding

\[ F_2(z) = \frac{A_T(z) - A_1(z)}{\lambda_2(z - 1)}. \]  \hspace{1cm} (11)

Using equations (4) and (11) in the \( z \)-transform of equation (8) eventually gives us the pgf of \( d_2 \), i.e.,

\[ D_2(z) = \frac{1 - \lambda_T}{\lambda_2} z(A_T(V(z)) - A_1(V(z))) \cdot \frac{V(z) - A_T(V(z))}{V(z) - A_T(V(z))}, \]  \hspace{1cm} (12)

where \( V(z) \) is implicitly determined by equation (10).

### 5 Calculation of moments

The functions \( Y(z) \) and \( V(z) \) can only be explicitly found in case of some simple arrival processes. Their derivatives for \( z = 1 \), necessary to calculate the moments of the system contents and the cell delay, on the contrary, can be calculated in closed-form. For example, the first derivatives are given by

\[ Y'(1) = \frac{\lambda_2}{1 - \lambda_1}, \quad V'(1) = \frac{1}{1 - \lambda_1}. \]
Let us define \( \lambda_{ij} \) as

\[
\lambda_{ij} \triangleq \left. \frac{\partial^2 A(z_1, z_2)}{\partial z_i \partial z_j} \right|_{z_1 = z_2 = 1},
\]

with \( i, j = 1, 2 \). Now we can calculate the mean values of the different system contents and cell delays by taking the first derivative of the respective pgf's for \( z = 1 \). We find

\[
E[u_T] = \lambda_T + \frac{A''(1)}{2(1 - \lambda_T)},
\]  

(13)

for the mean value of total system contents,

\[
E[u_1] = \lambda_1 + \frac{\lambda_{11}}{2(1 - \lambda_1)},
\]  

(14)

for the mean value of system contents of class 1 cells and

\[
E[u_2] = \lambda_2 + \frac{2\lambda_{12} + \lambda_{22}}{2(1 - \lambda_T)} + \frac{\lambda_2 \lambda_{11}}{2(1 - \lambda_T)(1 - \lambda_1)},
\]  

(15)

for the mean value of the system contents of class 2 cells. It is easily verified that equations (13), (14) and (15) satisfy \( E[u_T] = E[u_1] + E[u_2] \).

Furthermore, from equations (7) and (12), we derive the following expressions

\[
E[d_1] = 1 + \frac{\lambda_{11}}{2\lambda_1(1 - \lambda_1)}
\]  

(16)

and

\[
E[d_2] = 1 + \frac{2\lambda_{12} + \lambda_{22}}{2\lambda_2(1 - \lambda_T)} + \frac{\lambda_{11}}{2(1 - \lambda_T)(1 - \lambda_1)},
\]  

(17)

for the mean value of the cell delay of a class 1 and a class 2 cell respectively. We can see from equations (14) - (17) that Little's law \( E[u_j] = \lambda_j E[d_j] \) \( (j = 1, 2) \) is fulfilled for both classes, as expected.

In a similar way, expressions for the variance of system contents and cell delay and some interesting correlation coefficients can be calculated by taking the appropriate derivatives of the respective generating functions as well.

6 Discussion of the results and special relations

In this section, we will discuss some of the results from the former two sections. First, we notice that the pgf of the total system contents (equation (4)) is the same as for a single class system with an identical cell arrival process described by \( A_T(z) \). Indeed, since the service time is deterministic and equal to 1 slot for the two classes, the scheduling is not important in terms of total system contents.

Second, we see that the system contents of class 1 cells is not influenced by class 2 cells and furthermore that its pgf has the same structure as \( U_T(z) \). This is of course due to the HOL
priority scheduling. For class 1 cells, it seems as if no class 2 cells are present in the system. Consequently, since the scheduling is FIFO within class 1, \( U_1(z) \) and \( D_1(z) \) fulfill the following relation (see [16]):

\[
U_1(z) = 1 - \lambda_1 + \lambda_1 D_1(z).
\]

It is easily verified that (5) and (7) satisfy this equation.

In the special case that the arrivals of class 1 and class 2 cells are uncorrelated, i.e., \( A(z_1, z_2) = A_1(z_1)A_2(z_2) \), we can calculate the system contents of class 2 in an alternative way. Since class 2 cells can only be served when there are no class 1 cells in the system, we can model the system, with respect to class 2 cells, in terms of a system with server interruptions. The server is interrupted for class 2 cells if there are class 1 cells waiting to be sent, and it is available if there are not. We can then calculate the pgf of the duration of busy and idle period of class 1 cells, i.e., the time period during which there are class 1 cells in the system (i.e., \( u_1 > 0 \)) and the time period during which there are no such cells (i.e., \( u_1 = 0 \)), respectively. It is easily verified that the duration of the idle period is geometrically distributed, i.e., its pgf is given by

\[
I(z) = \frac{(1 - A_1(0))z}{1 - A_1(0)z}.
\]

The calculation of the busy period is a bit more involved, and can be found in [3] for a general service time distribution. In case of a deterministic service time of one slot, it is implicitly given by the following formula:

\[
B(z) = \frac{A_1(z((1 - A_1(0))B(z) + A_1(0))) - A_1(0)}{1 - A_1(0)}.
\]

It is clear that when the system is busy with respect to class 1 cells, it is interrupted for class 2 cells. So \( I(z) \) and \( B(z) \) are also the pgfs of the time periods during which the server is available and the time periods during which the server is interrupted for class 2 cells, respectively. Furthermore, the lengths of consecutive idle and busy periods are statistically independent. In [2], such a system is analyzed. Using those results and defining \( X(z) \) as \( A_2(z)(A_1(0) + (1 - A_1(0))B(A_2(z))) \) leads to the following pgf for the system contents of class 2 cells

\[
U_2(z) = (1 - \lambda_2) \frac{1 - X(z) A_2(z)(z - 1)}{1 - A_2(z)} z - X(z),
\]

with

\[
X(z) = A_1(X(z))A_2(z).
\]

Equation (6) and (18) lead to the same result for \( U_2(z) \), when \( X(z) = Y(z) \). This is the case when class 1 and class 2 arrivals during a slot are uncorrelated.

7 Application

In this section, we apply our results from the former sections to an ATM output-queueing switch. We consider a non-blocking switch with \( N \) inlets and \( N \) outlets. We assume two types
of traffic. Traffic of class 1 is delay-sensitive (for instance voice) and traffic of class 2 is assumed to be delay-insensitive (for instance data). We investigate the effect of HOL priority scheduling, as presented in the former of this paper.

The cell arrivals on each inlet are assumed to be i.i.d., and generated by a Bernoulli process with arrival rate $\lambda_T$. An arriving cell is assumed to be of class $j$ with probability $\lambda_j/\lambda_T$ ($j = 1, 2$) ($\lambda_1 + \lambda_2 = \lambda_T$). The incoming cells are then routed to the output queue corresponding to their destination, in an independent and uniform way. Therefore, the output queues behave identically and we can concentrate on the analysis of 1 output queue. In view of the previous, the arrivals of both types of cells to an output queue are generated according to a twodimensional binomial process. It is fully characterized by the following joint pgf

$$A(z_1, z_2) = (1 - \frac{\lambda_1}{N}(1 - z_1) - \frac{\lambda_2}{N}(1 - z_2))^N.$$ 

Obviously, the number of class 1 and class 2 arrivals at an output queue during a slot are correlated. This is simply demonstrated by the following observation: when $m$ class 1 cells arrive at the tagged queue during a slot ($0 \leq m \leq N$), the maximum number of class 2 arrivals during the same slot is limited by $N - m$. We note that for $N$ going to infinity, the above expression becomes a product of two generating functions of Poisson distributions with means $\lambda_1$ and $\lambda_2$ respectively, and as a result, the arrival process becomes uncorrelated for both classes. In the following, we will investigate the effect of priority scheduling on some performance measures, such as mean value and variance of system contents and cell delay. We will, when possible, compare with a FIFO scheduling to show the advantages and disadvantages of priority scheduling.

In the remaining of this section, we assume a 16x16 switch ($N = 16$). We analyze the performance of the switch for three special cases, namely when the fraction of class 1 cells that arrive at the inlets is 0.25, 0.5 and 0.75 respectively.

In Figures 2 and 3, mean value and variance of the system contents of class 1 and class 2 cells is shown as a function of the total load. One can easily see the influence of priority scheduling: the mean, as well as the variance of the number of class 1 cells in the system is severely reduced to the HOL scheduling; the opposite holds for class 2 cells. It is also clear that the impact of priority scheduling is more important if the load is high.

In Figure 4, the correlation coefficient $\rho_{u_1u_2}$, which quantifies the correlation between the number of class 1 and class 2 cells in the system during a slot, is shown as a function of the total load. We see that $\rho_{u_1u_2}$ increases when the fraction of class 1 cells increases (for a given total load). This can easily be understood by the priority scheduling. The influence of class 1 cells on class 2 cells will become more important, when the fraction of class 1 cells increases. A second observation is that $\rho_{u_1u_2}$ is (slightly) negative when the total load is small, but becomes positive when the total load is large. The reason for this are two counteracting mechanisms. The first one is the switch structure. When more class 1 cells arrive at the switch, there will be less class 2 cells arriving at the same time (since the amount of inlets is limited), and vice versa. This negative correlation between cell arrivals during a slot shows for small values of $\lambda_T$. The second influence is priority scheduling. The larger $\lambda_1$, the larger the system contents of class 1 cells, and the larger the impact on class 2 cells, thus a positive correlation coefficient. Finally, when $\lambda_T$ approaches 1, the total system contents (and the number of class 2 cells) approaches infinity, due to the system becoming unstable rather than the priority scheduling. As a result $\rho_{u_1u_2}$ approaches 0.

Figures 5 and 6 show the mean value and the variance of cell delay as a function of the total load for the three cases mentioned above. In order to compare with FIFO scheduling, we have also shown the mean value and variance of the cell delay in that case. The cell delay is then of
course the same for class 1 and class 2 cells, and can thus be calculated as if there is only one class arriving according to an arrival process with pgf \( A(z,z) \). This has already been analyzed, e.g., in [5] for the special case of a multiserver output-queuing switch. We observe that the influence of HOL priority scheduling is quite large, especially when the arrival rate of class 1 cells is not too large. Mean delay and delay-jitter of class 1 cells reduces considerably compared to FIFO scheduling. The price to pay is of course a bigger mean delay and delay-jitter for class 2 cells. If this kind of traffic is not delay-sensitive, this is not too big a problem. Nevertheless, in a buffer with limited storage, an appropriate space priority scheme will have to be applied in order to avoid excessive cell loss of class 2 cells.
Figure 4: Correlation of system contents when the fraction of class 1 arrivals equals 0.25, 0.5 and 0.75

Figure 5: Mean value of cell delays when the fraction of class 1 arrivals equals 0.25, 0.5 and 0.75

8 Conclusions

In this paper, we analyzed a queueing system with HOL priority scheduling. A generating-functions approach was adopted, which led to closed-form expressions of performance measures, such as mean and variance of the system contents and cell delay, and the correlation coefficient of the system contents of both types of cells, that are easy to evaluate. The model included possible correlation between the number of arrivals of the two cell types during a slot. Therefore, the results could be used to evaluate the performance of a prioritized output-queueing switch with Bernoulli arrivals.
Figure 6: Variance of cell delays when the fraction of class 1 arrivals equals 0.25, 0.5 and 0.75

References


