**Sustainable load of fibre delay line buffers**

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The sustainable load of fibre delay line buffers, defined as the load at which a system with infinite buffering capacity becomes unstable, is discussed. Owing to the particularities of fibre delay line buffers, namely their finite delay granularity, this sustainable load is generally less than 100%. It is further shown that the packet-size distribution has some impact too.

**Introduction:** In packet switching, contention inevitably output port needs to be resolved. One option is to buffer one (or more) of the packets in conflict, until the moment they can be safely transmitted. In the electronic domain, the use of electronic random access memory (RAM) allows packets to be stored for an arbitrary period of time, which in turn allows efficient scheduling on the output channel, without waste of capacity. Currently, this is not the case in the optical domain, owing to the lack of optical RAM. In the foreseeable future, fibre delay lines (FDLs) seem to be the most viable alternative. In the time domain, owing to the lack of optical RAM. In the foreseeable future, which in turn allows efficient scheduling on the output channel, avoidance of output port contention. The best one can do is to delay the packet until the next available time slot.

A quantity of interest in the study of FDL-based buffers is the scheduling horizon [1, 2]. It is the earliest time, pending new arrivals, at which the channel will become available again. If new arrivals, at which the channel will become available again. If

\[ D \equiv \frac{B_k}{C} \]

is the mean packet size, \( B_z \) is the generating function \( E[z^N] \) of the packet-size distribution, and \( p_{max} = 1 - \rho_{max} \). The sum involves \( z_k \), the \( D \) different complex 2-th order roots of unity (where, by definition, \( z_k = 1 \)). A first approximation, in which only \( E[B] \) appears, follows if one neglects the contribution of that sum:

\[ p_{max} \approx \left( E[B] + \frac{D - 1}{2} \right)^{-1} \]

(2)

This approximation also follows by an intuitive reasoning [1]: each packet contributes to the scheduling horizon by its own size, \( E[B] \), and by a so-called void, created due to its suboptimal scheduling in a finite-granularity buffer. Assuming this void is uniformly distributed on the set \( \{0, \ldots, D - 1\} \) (recall we are studying the problem in a slotted-time setting), the average void is \( (D - 1)/2 \). The term \( E[B] + (D - 1)/2 \) can then be interpreted as an average equivalent packet size, in which void creation is accounted for. Approximation (2) then easily follows.

The exact relation (1) depends on the details of the packet-size distribution, here through

\[ B(z_k) = \sum_{n=0}^{\infty} e_n^p \Pr[B = n] \]

Thus only the distribution of \( B \) mod \( D \) plays a role. Let us therefore introduce the quantities

\[ \hat{b}(n) = \sum_{m=0}^{\infty} \Pr[B = mD + n] \]

(Defining these \( \hat{b}(n) \) for \( 1 \leq n \leq D \), instead of for \( 0 \leq n \leq D - 1 \), somewhat simplifies the results to be presented next.) One can then rewrite (1) in the following way:

\[ E\left[ \frac{B}{D} \right] \geq \frac{\sum_{n=1}^{D} \hat{b}(n) p_{max}}{1 - p_{max}} \]

(3)

Once all parameters of interest are determined, numerical solution of this equation for \( p_{max} \) is straightforward.

**Special cases:** Some packet-size distributions allow for further simplification of (3). If, for instance, packet lengths are uniformly distributed on \( \{1, \ldots, K \cdot D\} \) for some \( K \geq 1 \), one can show that (2) becomes exact. However, when packet sizes are deterministic of size \( B \), we can always write \( B = a \cdot D + c \), with \( a \geq 0 \) and \( 0 \leq c \leq D \), and one easily finds

\[ E\left[ \frac{B}{D} \right] = a + 1 \]

\[ \hat{b}(n) = \delta_{n,a} \quad 1 \leq n \leq D \]
and

$$a + 1 = \frac{p_{\text{max}}}{1 - p_{\text{max}}}$$

It is a simple exercise to establish that, for given $a$ and $D$, the case $c = 1$ results in the highest $p_{\text{max}}$ value. Reversing the argument, this shows that $p_{\text{max}}$ as a function of $D$, will exhibit ‘local maxima’ whenever $D \geq (B - 1)/a$ ($a$ a integer).

Finally, if packet sizes follow a geometric distribution, $\Pr[B = n] = \alpha(1 - \alpha)^{n-1}$ for $n \geq 1$ (such that $E[B] = 1/\alpha$), (3) reduces to

$$1 = \frac{\alpha}{1 - p_{\text{max}}} \cdot \frac{1 - (2p_{\text{max}})^D}{1 - 2p_{\text{max}}}$$

where $\alpha = 1 - \alpha$. For typical parameter values, i.e. $1 \ll D/2 < 1/\alpha$, (2) becomes sufficiently accurate once more.

### Numerical example:

In Fig. 2, we have plotted $\rho_{\text{max}} = p_{\text{max}} E[B]$ against $D$ for deterministic packet sizes of size 100, and also the approximation given in (2). The ‘local maxima’ $D \geq (B - 1)/a$ for the deterministic case clearly show (at $D = 99, 49, 33, \ldots$). The curve (not shown in Fig. 2) for geometrically distributed packet sizes, with mean $E[B] = 100$, is indistinguishable from the approximation (see above).

Clearly, in terms of sustainable load, the granularity $D$ should be chosen as small as possible, i.e. $D = 1$. However, in reality, i.e. for finite FDL buffers, there is a trade-off to be made [1, 2]. A small $D$ implies many FDLs have to be used to arrive at a given capacity $N \cdot D$, or, vice versa, when $N$ is fixed, a small $D$ will lead to a small capacity, resulting in large loss figures. Conversely, a large $D$ requires fewer FDLs or realises a larger capacity, resulting in lower loss, but seriously impairs performance owing to the typically large size of the voids created due to suboptimal scheduling.

### Conclusions:

We have shown that, in general, infinite FDL buffers cannot sustain loads up to 100%, owing to their finite granularity. This granularity creates voids in the channel scheduling, wasting transmission capacity. An implicit formula for the sustainable load was derived, in which the details of the packet-size distribution appear. An approximation, based on intuitive reasoning, was shown to be good in most cases, exact in some special ones.

The arrival process considered here—IID geometric interarrival times and IID packet sizes—served only as a good first model. Currently, we are investigating the impact of correlation between (consecutive) interarrival times or (consecutive) packet sizes, and also that of cross-correlation between packet sizes and interarrival times. We hope to report on these results in the near future.

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**Fig. 2** Sustainable load against granularity $D$ ($E[B] = 100$)

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**References**
