Buffer contents and cell delay in a rate adaptation buffer with Markovian arrivals

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Received 1 December 1998

Abstract

This paper investigates the performance of a rate adaptation buffer in the case that the arriving cell stream is generated by an on/off-source, where both the on-periods and the off-periods are geometrically distributed. The ratio between the input rate and the output rate takes an arbitrary integer value greater than one. Under the assumption of an infinite storage capacity, exact explicit expressions are obtained for the mean values and the tail distributions of the buffer contents and the cell delay. Furthermore, an approximation is derived for the cell loss ratio in a finite-capacity buffer. Some numerical results are presented and discussed. © 2001 Elsevier Science Ltd. All rights reserved.

Scope and purpose

In communication networks a rate adaptation buffer is used at the interface between two consecutive links, when the speed of the incoming link exceeds the speed of the outgoing link, in order to avoid excessive loss of information. So far, only few papers in the literature have investigated the buffer dimensioning of a rate adapter. All of these papers assume an uncorrelated arrival process on the incoming link and/or small differences between the input and the output rates, assumptions which in practice may not always be realistic. The present paper therefore presents and analyzes a discrete-time queueing model for a rate adaptation buffer which both accounts for the presence of correlation in the arrival stream and allows (possibly) large input/output ratios.

Keywords: ATM; Rate adaptation; Correlated arrivals; Queueing

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1 SMACS: Stochastic Modeling and Analysis of Communication Systems
1. Introduction

In modern telecommunication systems, information sent by a source typically passes through a large number of nodes before reaching its final destination. In practical (multivendor) networks, the communication links that interconnect multiple subsystems do not necessarily operate at the same transmission speeds. Consequently, situations occur where at the interconnecting interface the access rate of the incoming information exceeds the amount of information the outgoing link can handle per time unit. This is for instance the case at the user-network interface (UNI) in an asynchronous transfer mode (ATM) network, where information arriving on a high-speed ATM link is demultiplexed into one or more low-speed cell streams destined for the end users. Even in the case where two end users communicate with the network through access lines of equal speed, contention at the destinations’ UNI will occur due to the cell delay variation introduced throughout the ATM network. In these cases a rate adaptation buffer is required at the interface between the two consecutive links in order to avoid excessive cell loss.

To the best of our knowledge, only few papers in the literature have investigated the performance of a rate adaptation buffer. Considering a Bernoulli arrival stream and any values of the input rate/output rate ratio, an approximate procedure for calculating the cell loss ratio (CLR) was developed in Rothermel and Michiel [1,2] presents results for the CLR which are accurate for small differences between the input and output rates (less than 2%) and in Inghelbrecht et al. [3], exact expressions are derived for the moments and the tail distributions of the buffer contents and the cell delay in an infinite-capacity rate adaptation buffer and the CLR in a finite buffer. In Steyaert and Bruneel [4], an approximate analysis is given for the case where the arrival process is the output of a discrete-time M/D/1 queue, and the input/output ratio is equal to $k/(k - 1)$ for some integer $k$.

The present paper considers a time-correlated arrival process on the incoming link of the rate adapter. This means that in general, the number of cell arrivals during an input slot may depend on the numbers of cell arrivals during the previous input slots. Specifically, a first-order Markovian arrival process is assumed. The transmission rate of the incoming link is $n$ times higher than the transmission rate of the outgoing link, for some integer value $n$ greater than one. The purpose of this paper is to provide an analytical queueing analysis of a rate adapter under the above modeling assumptions. The paper can be viewed as an extension of Inghelbrecht et al. [3], in the sense that the cell arrival stream is now time correlated.

The rest of the paper is organized as follows. Section 2 describes the system under study. In Section 3, under the assumption of an infinite storage capacity, a functional equation is established which completely describes the system behavior. An analytical technique to derive the probability generating function (pgf) of the buffer contents from this functional equation is described in Section 4. In Section 5, explicit expressions are obtained for the mean value and the tail distribution of the buffer contents. Furthermore, an approximation is given there for the cell loss ratio in a finite-capacity buffer. The characteristics of the cell delay are derived in Section 6. Some numerical examples are presented and discussed in Section 7. Finally, conclusions are given in Section 8.

2. Model description

We consider a rate adaptation buffer, which is used in an ATM network at the interconnecting interface between two consecutive links, when the speed of the incoming link exceeds the speed of
the outgoing link. As in most ATM-related discrete-time models, the time axis is divided into fixed-length time intervals, referred to as slots, where one slot suffices for the transmission of exactly one cell. In this paper, the length of a slot at the output of the rate adaptation buffer is assumed to be $n$ times larger than the length of a slot at the input, with $n$ an integer greater than one. Furthermore, we assume that the rate adaptation buffer has an infinite storage capacity and operates according to a first-in-first-out (FIFO) scheduling discipline.

In this paper, the cell arrival process on the incoming link of the rate adapter is time correlated. Specifically, we define an on-period on the input link as a number of consecutive input slots during which one cell arrival occurs. Similarly, an off-period on the input link is a number of consecutive input slots during which there is no cell arrival. We assume here that the lengths of the on-periods and the off-periods are two independent sets of independent geometrically distributed random variables with parameters $\alpha$ and $\beta$ respectively, i.e.,

\[
\text{Prob[on-period } = i \text{ slots]} = (1 - \alpha)\alpha^{i-1}, \quad i \geq 1, \quad (1)
\]

\[
\text{Prob[off-period } = i \text{ slots]} = (1 - \beta)\beta^{i-1}, \quad i \geq 1. \quad (2)
\]

Note that this assumption implies a first-order Markovian correlation in the arrival process on the inlet. The case of an uncorrelated Bernoulli arrival process on the input link corresponds to $\alpha + \beta = 1$. The mean number of cell arrivals $\sigma$ during an input slot is given by

\[
\sigma = \frac{1 - \beta}{2 - \alpha - \beta}. \quad (3)
\]

### 3. Derivation of a functional equation

Let us define the random variable $s_k$ as the system contents at the beginning of the $k$th output slot, i.e., the total number of cells in the buffer at the start of the $k$th output slot including the cell that will be transmitted during the $k$th output slot. Also, let us denote by $e_{k,i}$ ($1 \leq i \leq n$) the number of cell arrivals during the $i$th input slot of the $k$th output slot. Fig. 1 illustrates these definitions.

In view of the above description of the queueing model, it then follows that the system contents evolves according to the system equation

\[
s_{k+1} = (s_k - 1)^+ + \sum_{i=1}^{n} e_{k,i}, \quad (4)
\]

where the symbol $(\ldots)^+$ denotes max$(0, \ldots)$.

Note that because of the correlated arrival process the random variables appearing in Eq. (4) are not statistically independent of each other. However, from (1), (2) and (4) it is easily seen that the value of the vector $(s_k, e_{k-1,n})$ suffices to determine the probability distribution of the vector $(s_{k+1}, e_{k,n})$. Hence, the vector $(s_k, e_{k-1,n})$ constitutes a Markovian state description of the queueing system at the start of output slot $k$.

In order to analyze the queueing behavior, let us now define the function $P_k(z, x)$ as

\[
P_k(z, x) \triangleq E[z^{s_k}x_0^{1-\epsilon_k}x_1^{\epsilon_k}]. \quad (5)
\]
where $E[\ldots]$ denotes the expected value of the argument between the square brackets and

$$x = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}.$$

Using this definition and the system equation (4), we then obtain the function $P_{k+1}(z, x)$ as

$$P_{k+1}(z, x) = E \left[ z^{(s_k - 1)^*} \sum_{z=1}^{n} e_{k,i} x_0^{1 - e_{k,i} x_1^{s_k}} \right],$$

which can be rewritten as

$$P_{k+1}(z, x) = E \left[ \sum_{z=1}^{n} e_{k,i} x_0^{1 - e_{k,i} x_1^{s_k}} \bigg| s_k = 0 \right] \text{Prob}[s_k = 0]$$

$$+ \sum_{j=1}^{\infty} z^{j-1} E \left[ \sum_{z=1}^{n} e_{k,i} x_0^{1 - e_{k,i} x_1^{s_k}} \bigg| s_k = j \right] \text{Prob}[s_k = j].$$

(6)

Next, if we define $\gamma_{k,j}(z, x)$ as

$$\gamma_{k,j}(z, x) \triangleq E \left[ \sum_{z=1}^{n} e_{k,i} x_0^{1 - e_{k,i} x_1^{s_k}} \bigg| s_k = j \right],$$

(7)

Eq. (6) can be transformed into

$$P_{k+1}(z, x) = z^{-1} \sum_{j=0}^{\infty} z^j \gamma_{k,j}(z, x) \text{Prob}[s_k = j] + (1 - z^{-1}) \gamma_{k,0}(z, x) \text{Prob}[s_k = 0].$$

(8)

In the appendix it is shown that $\gamma_{k,j}(z, x)$ equals

$$\gamma_{k,j}(z, x) = E[\beta_{0,n}(z, x)^{1 - e_{k,i} x_1^{s_k}} \beta_{1,n}(z, x)^{e_{k,i} x_1^{s_k}} | s_k = j],$$

(9)

where

$$\begin{pmatrix} \beta_{0,n}(z, x) \\ \beta_{1,n}(z, x) \end{pmatrix} \triangleq Q(z)^n \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

(10)

and

$$Q(z) \triangleq \begin{pmatrix} \beta & (1 - \beta)z \\ 1 - \alpha & \alpha z \end{pmatrix}.$$
Using result (9) and the property that $e_{k-1,n} = 0$, if $s_k = 0$, we then get the following equation for $P_{k+1}(z, x)$:

$$P_{k+1}(z, x) = z^{-1} \sum_{j=0}^{\infty} \sum_{l=0}^{1} z^j \beta_{0,n}(z, x)^{1-l} \beta_{1,n}(z, x)^l \text{Prob}[s_k = j, e_{k-1,n} = l]$$

$$+ (1 - z^{-1}) \beta_{0,n}(z, x) \text{Prob}[s_k = 0].$$

Finally, in view of definition (5), we obtain

$$P_{k+1}(z, x) = z^{-1} P_k(z, Q(z)^n x) + (1 - z^{-1}) \beta_{0,n}(z, x) \text{Prob}[s_k = 0].$$  \tag{12}

Let us now assume that the equilibrium condition, being the condition that the output load $\rho = n \sigma$ is strictly less than one, is satisfied so that the buffer system can reach a stochastic equilibrium. In this case, as $k \to \infty$, both the functions $P_k(z, x)$ and $P_{k+1}(z, x)$ will converge to a common limiting function

$$P(z, x) \triangleq \lim_{k \to \infty} P_k(z, x).$$

From (12), we find that $P(z, x)$ must satisfy the following functional equation:

$$P(z, x) = z^{-1} P(z, Q(z)^n x) + (1 - z^{-1}) \beta_{0,n}(z, x) p_0,$$  \tag{13}

where $p_0$ is the steady-state probability that the system contents at the beginning of an arbitrary output slot is equal to zero.

### 4. Solving the functional equation

In this section, we will present a technique to solve the functional equation (13). However, we start this section with some preliminary definitions and results. First, we define the $2 \times 1$ column matrix $A^\ast(z, x)$ as

$$A^\ast(z, x) \triangleq Q(z)^n x.$$  \tag{14}

Also, we introduce the set of functions, for $\nu \geq 1$ defined recursively as

$$A^\ast(z, x) \triangleq A^\ast(z, A^{\nu-1}(z, x)) = Q(z)^\nu x$$  \tag{15}

with

$$A^0(z, x) \triangleq x.$$  \tag{16}

For each value of $z$, we denote by $\lambda_i(z), 1 \leq i \leq 2$, the two eigenvalues of the matrix $Q(z)$, defined in (11). The eigenvalues $\lambda_1(z)$ and $\lambda_2(z)$ are given by

$$\lambda_1(z) = \frac{\beta}{2} + \frac{zx}{2} + \frac{((\beta + zx)^2 - 4z(x + \beta - 1))^{1/2}}{2},$$  \tag{17}

$$\lambda_2(z) = \frac{\beta}{2} + \frac{zx}{2} - \frac{((\beta + zx)^2 - 4z(x + \beta - 1))^{1/2}}{2}$$  \tag{18}
with the agreement that \((a e^{i\theta})^{1/2} = \sqrt{a} e^{i\theta/2}\) for all \(\theta > 0, 0 \leq \theta < 2\pi\). Furthermore, we denote by \([w_{ij}(z)]\) a 2 × 2 matrix for which the \(i\)th row is a left row eigenvector of \(Q(z)\) with eigenvalue \(\lambda_i(z)\), i.e.,

\[
w(z)Q(z) = \lambda(z)w(z)
\]

(19)

with

\[
w(z) = \begin{pmatrix} w_1(z) \\ w_2(z) \end{pmatrix} = \begin{pmatrix} w_{11}(z) & w_{12}(z) \\ w_{21}(z) & w_{22}(z) \end{pmatrix}
\]

and

\[
\lambda(z) = \begin{pmatrix} \lambda_1(z) & 0 \\ 0 & \lambda_2(z) \end{pmatrix}
\]

With \(u(z)\) being defined as the inverse matrix of \(w(z)\), Eq. (19) implies that

\[
Q(z)u(z) = u(z)\lambda(z),
\]

i.e., \(u(z)\) is a 2 × 2 matrix for which the \(j\)th column is a right column eigenvector of \(Q(z)\) with eigenvalue \(\lambda_j(z)\). In the following, it will become clear that it is preferable to define the left row eigenvectors of \(Q(z)\) such that

\[
\sum_{j=1}^2 w_{ij}(z) = 1, \quad i = 1, 2.
\]

(20)

The 2 × 2 matrix \(w(z)\) (and, consequently, its inverse \(u(z)\)) is uniquely determined by (19) and (20). Specifically, it is easily shown that the solution of (19) and (20) is given by

\[
w(z) = \begin{pmatrix} \frac{1 - \alpha}{1 - \alpha - \beta + \lambda_1(z)} & \frac{\lambda_1(z) - \beta}{1 - \alpha - \beta + \lambda_1(z)} \\ \frac{1 - \alpha}{1 - \alpha - \beta + \lambda_2(z)} & \frac{\lambda_2(z) - \beta}{1 - \alpha - \beta + \lambda_2(z)} \end{pmatrix}
\]

(21)

and hence,

\[
u(z) = \begin{pmatrix} (1 - \alpha - \beta + \lambda_1(z))z(1 - \beta) & (1 - \alpha - \beta + \lambda_2(z))z(1 - \beta) \\ (\lambda_2(z) - \alpha z)(\lambda_2(z) - \lambda_1(z)) & (\lambda_1(z) - \alpha z)(\lambda_1(z) - \lambda_2(z)) \end{pmatrix} \equiv \begin{pmatrix} u_{11}(z) & u_{12}(z) \\ u_{21}(z) & u_{22}(z) \end{pmatrix}
\]

(22)

Finally, based on the property that \(Q(z) = u(z)\lambda(z)w(z)\), the \(i\)th row element of the column vector \(A'(z, x)\), defined in Eq. (15), can be expressed as

\[
[A'(z, x)]_i = [u(z)\lambda(z)w(z)x]_i
\]

\[
\triangleq \sum_{l=1}^2 u_{il}(z)\lambda_i(z)x W_l(z, x), \quad i = 1, 2
\]

(23)
with

\[
\begin{pmatrix}
W_1(z, x) \\
W_2(z, x)
\end{pmatrix} \triangleq w(z)x. \tag{24}
\]

Let us now concentrate on solving the functional equation (13). From (13), with the definitions (15) and (16), we obtain the following relationship:

\[
z^{-(\nu-1)}P(z, A^{\nu-1}(z, x)) = z^{-\nu}P(z, A^{\nu}(z, x)) + z^{-(\nu-1)}(1 - z^{-1})\beta_{0,\nu}(z, A^{\nu-1}(z, x))P_0
\]

for all \(\nu \geq 1\). Summation of the above equation for consecutive values of \(\nu\) then yields

\[
P(z, x) = z^{-\nu}P(z, A^{\nu}(z, x)) + \sum_{k=1}^{\nu} z^{-(\nu-1)}(1 - z^{-1})\beta_{0,\nu}(z, A^{k-1}(z, x))P_0
\]

or with (5) and (10),

\[
P(z, x) = P\left(z, \frac{A^{\nu}(z, x)}{z^\nu}\right) + P_0 \sum_{k=1}^{\nu} \beta_{0,\nu}\left(z, \frac{A^{k-1}(z, x)}{z^k} (z - 1)\right) \tag{25}
\]

for any value of \(\nu \geq 1\).

We now consider values of \(z\) for which \(|\lambda_l(z)| < |z| \leq 1\), \(l = 1, 2\). Such values of \(z\) exist; in Steyaert and Bruneel [5], this inequality is proved to hold for all \(\{z : |z| = 1\text{ and } z \neq 1\}\). Then, in view of expression (23), taking the limit for \(\nu \to \infty\), the sum for \(k\) on the right-hand side of expression (25) converges, and we get

\[
P(z, x) = \sum_{k=1}^{\infty} \beta_{0,\nu}\left(z, \frac{A^{k-1}(z, x)}{z^k} (z - 1)\right)P_0,
\]

where we have used the property that \(P(z, 0, 0) = 0\). Working out the sum for \(k\) in the above formula, we finally find the following expression for the function \(P(z, x)\):

\[
P(z, x) = (z - 1) \sum_{j=1}^{2} \frac{\lambda_j(z)^n}{z - \lambda_j(z)^n} W_j(z, x)u_{1, j}(z)P_0. \tag{26}
\]

We are mainly interested in \(S(z)\), being defined as the pgf of the random variable \(s\), the system contents at the beginning of an arbitrary output slot in the steady state. Because \(S(z)\) equals \(P(z, 1, 1)\), setting \(x_0 = x_1 = 1\) in (26), and making use of (20) and (23), we obtain

\[
S(z) = (z - 1) \sum_{j=1}^{2} \frac{\lambda_j(z)^n}{z - \lambda_j(z)^n} u_{1, j}(z)P_0. \tag{27}
\]

The above expression still contains the unknown \(P_0\). This unknown probability can be calculated from the normalization condition \(S(1) = 1\), which yields

\[
p_0 = 1 - n\lambda_1'(1) = 1 - n\sigma = 1 - \rho, \tag{28}
\]
where $\rho$ is the output load. Note that this result is in full agreement with the property that in the steady state, the mean number of cell arrivals during an arbitrary output slot equals the mean number of cell departures during an output slot owing to the unlimited storage capacity.

5. System contents

From Eq. (27), all the important characteristics of the system contents, such as the mean value and the tail distribution, and even an approximation for the cell loss ratio in case of a finite-capacity buffer, can be derived. Specifically, we can calculate the mean system contents by taking the first derivative of the pgf $S(z)$ with respect to $z$ in the point $z = 1$. As a result, we get

$$E[s] = S'(1) = n\sigma + \frac{\sigma(1 - z - \beta)}{2 - z - \beta} + \frac{n(n - 1)\sigma^2 + 2n\sigma(1 - \sigma)((x + \beta - 1)/(2 - z - \beta))}{2(1 - n\sigma)}. \quad (29)$$

As has been indicated in for instance [6,7], the tail distribution of the system contents can be approximated very accurately by a geometric form. That is, for sufficiently large values of $S$, the probability that the system contents exceeds $S$ is given by

$$\text{Prob}[s > S] \approx - b_0 \frac{z_0^{-s-1}}{z_0 - 1}, \quad (30)$$

where $z_0$ is the real positive pole of $S(z)$ with the smallest modulus and the constant $b_0$ is the residue of $S(z)$ in the point $z = z_0$. It has been observed that $z_0$ is the real solution of the equation

$$z - \lambda_1(z)^n = 0$$

outside the unit disk with the smallest modulus. From expression (27) for $S(z)$, using de l'Hospital's rule, we obtain the residue $b_0$ as

$$b_0 = \lim_{z \to z_0} (z - z_0)S(z) = \frac{(1 - n\sigma)(z_0 - 1)\lambda_1(z_0)^n u_{11}(z_0)}{1 - n\lambda_1(z_0)^n - 1\lambda_1'(z_0)},$$

where $\lambda_1'(z_0)$ is given by

$$\lambda_1'(z_0) = \frac{x}{2} + \frac{(\beta + xz_0)x - 2(x + \beta - 1)}{2\sqrt{(\beta + xz_0)^2 - 4z_0(x + \beta - 1)}}.$$

From the above equations it is clear that the numerical evaluation of the tail distribution of the system contents is easy, since it merely requires the calculation of $z_0$, which can be done for instance by means of the Newton–Raphson algorithm.

Based on the tail distribution of the system contents it is also possible to derive an approximation for the cell loss ratio (CLR) in a finite-capacity buffer, i.e., the fraction of the arriving cells that is lost upon arrival because of buffer overflow. In order to do so, we note that in [3], under the assumption of an uncorrelated Bernoulli arrival process on the input link, it has been shown that
the CLR in a finite rate adaptation buffer of size $L$ is exactly given by

$$\text{CLR} = \frac{(1 - \rho) \text{Prob}[s > L]}{\rho(1 - \text{Prob}[s > L])},$$

(31)

where $\text{Prob}[s > L]$ is the probability of having a system contents larger than $L$ in the corresponding infinite-capacity buffer with the same arrival process and the same input rate/output rate ratio. In the case of a correlated arrival process, as in the present paper, formula (31) does no longer hold exactly, but we can use it to approximate the CLR. The accuracy of this approximation will be confirmed in Section 7 by some numerical examples.

6. Cell delay

We define the delay $d$ of a cell as the number of output slots between the end of the output slot during which the cell has arrived in the buffer and the departure instant of this cell, i.e., the end of the output slot during which the cell is transmitted from the buffer. In Bruneel and Kim [8] and Vinck and Bruneel [9], a general relationship was established between the distribution of the cell delay $d$ and the distribution of the system contents $s$, which is valid for any discrete-time G-D-1 queueing system with a FIFO queueing discipline, irrespectively of the (possibly correlated) nature of the cell arrival process. In terms of generating functions, this relationship reads:

$$D(z) = \frac{S(z) - S(0)}{1 - S(0)},$$

(32)

where $D(z)$ and $S(z)$ are the pgf’s of $d$ and $s$, respectively. From (32), the mean and the tail distribution of the cell delay can be derived as

$$E[d] = \frac{E[s]}{\rho},$$

(33)

$$\text{Prob}[d > D] = \frac{\text{Prob}[s > S]}{\rho}, \quad D \geq 1,$$

(34)

in terms of the characteristics of the system contents, derived above (see Eq. (29) and (30)). The analysis technique presented in this paper therefore also provides us with explicit expressions for the mean and the tail distribution of the cell delay. Note that (31) is nothing else than Little’s theorem [10].

7. Numerical results and discussion

An interesting parameter is the burstiness factor $K$, which is defined as the ratio between the average length of an on-period (or, equivalently, off-period) in the considered on/off-source model and the average length of an on-period (off-period) in the case of a Bernoulli arrival process with the same load $\sigma$. Hence, $K = 1$ corresponds to the case of an (uncorrelated) Bernoulli arrival
From the previous, we find that $K$ is given by

$$K = \frac{1 - \sigma}{1 - \alpha} = \frac{\sigma}{1 - \beta}. \quad (35)$$

It is clear that for a given value of $\sigma$, higher values of the parameter $K$ correspond to longer on-periods and off-periods, and therefore a higher “burstiness” or “irregularity” in the arrival process. In the following, we will characterize the cell arrival process in the rate adapter by the parameters $(\sigma, K)$ instead of $(\alpha, \beta)$.

In Fig. 2, the mean system contents $E[s]$ is shown versus the output load $\rho = n\sigma$ of the rate adaptation buffer, for $K = 3$ and various values of $n$. We observe that for a given value of the output load, the mean system contents increases as the bit-rate ratio $n$ increases and approaches to some limiting value for large values of $n$. This can be understood intuitively by the fact that the variance of the number of cells arriving in an output slot $\text{var}[\varepsilon] = \rho(1 - \rho/n)$ also increases with $n$ and goes to $\rho$ in the limit for $n \to \infty$.

In Fig. 3, we have plotted the mean system contents $E[s]$ versus the burstiness factor $K$, for $\rho = 0.8$ and various values of $n$. From this figure, we see that for given values of $\rho$ and $n$, the mean system contents is a linearly increasing function of the burstiness factor $K$. Very similar curves can also be obtained for the mean cell delay. The above observation is in full accordance with expression (29) for $E[s]$. Indeed, if we rewrite (29) in terms of $\rho$ and $K$, we obtain

$$E[s] = \rho + \frac{\rho}{n}(1 - K) + \frac{\rho^2(1 - 1/n) + 2\rho(1 - \rho/n)(K - 1)}{2(1 - \rho)}, \quad (36)$$

which is a linear function of $K$.

In Fig. 4, we compare the exact value of the CLR in a finite buffer of size $L$, obtained by direct numerical calculation, with approximation (31), for $\rho = 0.8$, $n = 2$ and various values of the burstiness factor $K$. The figure reveals that for low values of $K$, the approximation for the CLR nearly coincides with the exact CLR, which could be expected, since (31) is exact for $K = 1$. 

Fig. 2. Mean system contents $E[s]$ versus the load $\rho$, for $K = 3$ and $n = 2, 4, 16, 256.$
Furthermore, we notice that for all values of the system parameters, the approximation constitutes an upper bound for the exact value of the CLR, which is quite close. Finally, the figure also illustrates that the cell loss ratio increases as the cell stream becomes more bursty.

Fig. 5 shows the exact CLR and approximation (31) for the CLR versus the buffer size $L$, for $\rho = 0.8$, $K = 4$ and various values of $n$. The figure first of all confirms the accuracy of the approximation for the CLR. Secondly, it is clear from this figure that the buffer requirements in the rate adapter are increasing with $n$.

8. Conclusions

In this paper, we have evaluated the performance of a rate adaptation buffer fed by a bursty on/off-source with geometrically distributed on-periods and off-periods. The transmission rate of
the incoming link is $n$ times higher than the rate of the outgoing link, where $n$ is assumed to be an integer. By means of a generating-functions approach, exact closed-form expressions for the means and the tail distributions of the system contents and the cell delay in the case of an infinite storage capacity, as well as an accurate approximation for the cell loss ratio in case of a finite buffer size have been derived. Also, numerical results have been presented in order to investigate the influence of the burstiness of the arrival process and the ratio between the input rate and the output rate on the performance of the rate adapter.

Future research might consider the case where the input rate/output rate ratio can take any rational value greater than one.

**Acknowledgements**

The fourth author is postdoctoral fellow of the Fund for Scientific Research - Flanders (Belgium) (F.W.O.).

**Appendix. Proof of formula (9) for $\gamma_{k,j}(z, x)$**

From definition (7) of $\gamma_{k,j}(z, x)$, we have

$$\gamma_{k,j}(z, x) = E\left[\sum_{z=1}^{n} e_{k,z}X_0^{1-\epsilon_{k,z}X_0^{n}}|s_k = j\right]$$

$$= \sum_{l_0=0}^{1} \sum_{l_1=0}^{1} \cdots \sum_{l_n=0}^{1} \text{Prob}[e_{k-1,n} = l_0, e_{k,1} = l_1, \ldots, e_{k,n} = l_n|s_k = j]$$
\[ E \left[ \sum_{i=0}^{n} e_{i,n} - e_{i-1,n} | e_{k-1,n} = l_0, e_{k,1} = l_1, \ldots, e_{k,n} = l_n, s_k = j \right] \]

\[ = \sum_{l_0=0}^{1} \text{Prob}[e_{k-1,n} = l_0 | s_k = j] \cdot \left( \sum_{l_1=0}^{1} \cdots \sum_{l_n=0}^{1} \text{Prob}[e_{k,1} = l_1 | e_{k-1,n} = l_0] \cdot \text{Prob}[e_{k,2} = l_2 | e_{k,1} = l_1] \cdots \text{Prob}[e_{k,n} = l_n | e_{k,n-1} = l_{n-1}] \cdot z^{l_1 + \cdots + l_n} x_0^{1-l_n} x_1^{l_n} \right) \]

\[ = \beta_{0,n}(z, x)^{1-l_n} \beta_{1,n}(z, x)^{l_n} \]

By means of induction, it is now possible to show that

\[ \sum_{l_1=0}^{1} \cdots \sum_{l_n=0}^{1} \text{Prob}[e_{k,1} = l_1 | e_{k-1,n} = l_0] \cdot \text{Prob}[e_{k,2} = l_2 | e_{k,1} = l_1] \cdots \text{Prob}[e_{k,n} = l_n | e_{k,n-1} = l_{n-1}] \cdot z^{l_1 + \cdots + l_n} x_0^{1-l_n} x_1^{l_n} = \beta_{0,n}(z, x)^{1-l_n} \beta_{1,n}(z, x)^{l_n} \]

where

\[ \left( \begin{array}{c} \beta_{0,n}(z, x) \\ \beta_{1,n}(z, x) \end{array} \right) = Q(z)^n \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \]

and

\[ Q(z) = \begin{pmatrix} \beta & (1-\beta)z \\ 1-z & z \end{pmatrix}. \]

Substitution of (A.2) in (A.1) then gives

\[ \gamma_{k,j}(z, x) = \sum_{l_0=0}^{1} \text{Prob}[e_{k-1,n} = l_0 | s_k = j] \beta_{0,n}(z, x)^{1-l_n} \beta_{1,n}(z, x)^{l_n} \]

\[ = E[\beta_{0,n}(z, x)^{1-e_{k-1,n}} \beta_{1,n}(z, x)^{e_{k-1,n}} | s_k = j]. \]

References


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