Buffering for transmission rate reduction by a rational factor

H. Bruneel, V. Ing Hellobrrecht and B. Steyaert

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In ATM networks, cell loss may occur when transmission links of unequal speeds are interconnected. This problem can be alleviated by means of a rate adaptation buffer. The authors present an exact queuing analysis of such a buffer, valid for any rational input/output speed ratio.

Introduction: A rate adaptation buffer is used in communication networks at the interface between two consecutive links, when the speed of the incoming link exceeds the speed of the outgoing link, in order to avoid excessive cell loss [1]. In [2] we examined the performance of such a buffer in the case that the ratio between the input rate and the output rate takes an integer value. In this Letter we present a more general exact analysis, whereby the ratio between input and output rate can take any rational value greater than one. Among other things, the results allow an evaluation of storage requirements, and delay and loss characteristics of rate adapters in case of a small mismatch between input and output rates.

Description of model: We assume that the transmission rate of the incoming link is \( r \) times higher than the transmission rate of the outgoing link for some rational value \( r > 1 \). This means that we have the following relationship between the input and output rates: \( I = n \cdot O + m \), where \( I \) and \( O \) are the input and output rates, both expressed as an integer number of cells per fixed time interval, \( n \) is an integer greater than one and \( m \) is an integer between 0 and \( O - 1 \). As in most ATM-related discrete-time models, the time axis is assumed to be divided into slots of fixed length, where one slot suffices for the transmission of exactly one cell. Here, however, owing to the assumed rate difference between input and output, the length of a slot at the output of the rate adaptation buffer is greater than the length of a slot at the input. A time interval composed of \( I \) input slots or \( O \) output slots will be called a 'period' in the sequel. In this way, a period at the input of the buffer has the same length as a period at the output. We assume that input and output periods occur synchronously (without phase shift), i.e. have exactly the same sequence of starting points. The arriving cell stream on the incoming link of the rate adapter is modelled as a Bernoulli process. We assume that during each input slot, a cell is carried with probability \( p \) (i.e. \( p \) is the load of the input link), and that arrivals occur independently from slot to slot. Finally, we assume that the size of the rate adaptation buffer is infinite and we consider a FIFO (first-in, first-out) queuing discipline.

Buffer contents: We call \( S_k \) (with probability generating function (PGF) \( S_k(z) \)) the buffer contents at the beginning of the \( k \)th output slot \((1 \leq k \leq O)\) of the \( k \)th period of the timescale. We call \( A_k \) (with PGF \( A_k(z) \)) the number of arrivals during the \( k \)th input slot of the \( k \)th period of the timescale. Let \( n_t \) denote the maximum number of cells that can enter the buffer during the \( k \)th output slot of a period, i.e. the number of input slot boundaries that fall within the duration of the \( k \)th output slot. We can see that \( n_t \) is given by

\[
n_t = \left\lfloor \frac{I}{O} \right\rfloor - \left\lfloor \frac{I}{O} (t - 1) \right\rfloor
\]

where \( \lfloor x \rfloor \) denotes the integer part of the real number \( x \). The buffer contents evolves according to

\[
\begin{align*}
S_{k+1}(z) &= \frac{S_k(z) - 1}{z} + \sum_{j=1}^{\left\lfloor \frac{I}{O} (t-1) \right\rfloor + 1} e_{k,j} z^{-j} \quad \text{for } 0 \leq k \leq O - 1
\end{align*}
\]

where \( e_{k,j} \) is the cell number of type \( j \) that enters the buffer during the \( k \)th input slot of the \( k \)th period of the timescale.

Introducing PGEs in these equations, we obtain

\[
\begin{align*}
S_{k+1}(z) &= E(z)^n S_k(z) + E(z) \frac{S_k(z) - 1}{z} + \sum_{j=1}^{\left\lfloor \frac{I}{O} (t-1) \right\rfloor + 1} e_{k,j} z^{-j} \quad \text{for } 0 \leq k \leq O - 1
\end{align*}
\]

where \( E(z) = 1 - p \cdot r \) is the PGE associated with the Bernoulli arrival process. Assuming stochastic equilibrium (for \( k \to \infty \)) we can suppress the \( k \)-dependence of the PGEs and solve this set of linear equations, which yields

\[
S_k(z) = \frac{z - 1}{z^O - E(z)^n} H_k(z)
\]

where \( H_k(z) \) is given by

\[
H_k(z) = \frac{1}{O} \sum_{j=1}^{\left\lfloor \frac{I}{O} (t-1) \right\rfloor + 1} \sum_{i=0}^{\left\lfloor \frac{I}{O} (t-1) \right\rfloor + 1} H_{k-1}(z) \frac{z^{-i} - z^{-j}}{z^O - E(z)^n} + \sum_{j=1}^{\left\lfloor \frac{I}{O} (t-1) \right\rfloor + 1} \sum_{i=0}^{\left\lfloor \frac{I}{O} (t-1) \right\rfloor + 1} H_{k-1}(z) \frac{z^{-i} - z^{-j}}{z^O - E(z)^n}
\]

The PGE \( S(z) \) of the buffer contents \( s \) at the beginning of an arbitrary output slot is given by the arithmetic mean of the \( S_k(z) \):

\[
S(z) = \frac{z - 1}{z^O - E(z)^n} \frac{1}{O} \sum_{k=0}^{O-1} H_k(z)
\]

The \( O \) unknown constants \( S(0) \) in the results follow from the analyticity of \( S(z) \) inside the unit disk of the complex \( z \)-plane and Rouche's theorem [3] which gives \( O \)-1 equations, and the normalisation condition which gives one equation. The corresponding mean buffer contents is given by the first derivative of \( S(z) \) in the point \( z = 1 \):

\[
E[s] = S'(1) = \frac{\frac{1}{O} \sum_{k=0}^{O-1} H_k(1)}{2(1 - p) + 1 - \frac{I}{O}}
\]

The variance of the buffer contents follows from

\[
\text{Var}[s] = S''(1) + S'(1)^2
\]

For large values of \( S \), the tail probability of the buffer contents can be accurately approximated (see e.g. [3]) as

\[
P(s > z) \approx -b_{z_0} b_{z_0}^{O-1} - b_{z_0}^O - 1
\]

where \( z_0 \) is the real positive pole of \( S(z) \) with the smallest absolute value, and \( b_{z_0} \) is the residue of \( S(z) \) at \( z = z_0 \) given by

\[
b_{z_0} = \frac{1}{O} \sum_{k=1}^{O} H_k(z_0)
\]

In [4], for GI-D-1 models, an exact expression for the cell loss ratio (CLR) in a buffer of size \( K \) is derived from the tail probabilities of the buffer contents in a corresponding infinite-capacity queue:

\[
\text{CLR}(K) = \frac{(1 - q) P(s > K)}{q(1 - P(s > K))}
\]

where \( q = p \cdot r \) is the mean number of arrivals in an output slot. Using this formula as an approximation in the present case, we find for the CLR in a buffer of size \( K \)

\[
\text{CLR}(K) \approx \frac{(1 - p \cdot r) b_{z_0} \cdot z_0^{K-1}}{p \cdot r [1 - b_{z_0} \cdot z_0^{K-1}]}
\]

which is exact for integer values of \( r \).

Cell delay: In [3], the following relationship between the PGEs of the cell delay (\( D(z) \)) and the buffer contents (\( S(z) \)) in discrete-time G-D-1 models was shown to hold irrespectively of the nature of the arrival process:

\[
D(z) = \frac{S(z) - 1 + p \cdot r}{p \cdot r}
\]

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Using this equation—which is indeed applicable in the model investigated here—we find the mean cell delay as

$$E[d] = D'(1) = \frac{E[s]}{p \cdot r}$$  \hspace{1cm} (14)

the variance of the cell delay as

$$\text{Var}[d] = \frac{S''(1)}{p \cdot r} + \frac{S'(1)^2}{(p \cdot r)^2}$$  \hspace{1cm} (15)

and the tail distribution of the cell delay as

$$P(d > D) = \frac{P(s > D)}{p \cdot r} \quad D \geq 1$$  \hspace{1cm} (16)

Some numerical results: As opposed to the study reported in [2] (valid only for integer values of the rate ratio $r$), the current analysis allows us to investigate the buffering requirements and cell delay and loss characteristics of a rate adapter in the case of a ‘small’ mismatch between the transmission rates of the incoming and outgoing links, i.e., for values of $r$ slightly higher than 1.

![Graph showing tail distribution of buffer contents and CLR in buffer of size $K$, for $r = 1.1$ and 1.2, and $p = 0.8$](image)

Some results are shown in Fig. 1. Specifically, Fig. 1 shows the tail probabilities of the buffer contents (from eqn. 9) and the CLR (from eqn. 12) in the same graph for $r = 1.1$ and $r = 1.2$, assuming an input load $p = 0.8$ in both cases. Very similar graphs can be obtained for the tail probabilities of the cell delay (using eqn. 16). As can be observed from the results, a considerable number of cells may accumulate in the rate adaptation buffer, if it is large enough, resulting in long cell delays, or in reverse, a dramatic cell loss ratio may occur at the interface between two links if the buffer can store just a few cells, or without buffer. In fact, the required buffer space in the rate adapter to guarantee a negligible cell loss ratio (say 10⁻⁸ or less) can be quite substantial, even for relatively ‘small’ deviations between incoming and outgoing transmission rates.

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H. Brunelle, G. Inghelbrecht and B. Steyaert (Stochastic Modeling and Analysis of Communication Systems (SMACS) Research Group, Laboratory for Communications Engineering, University of Ghent, Sint-Pietersnieuwstraat 41, B-9000 Gent, Belgium)

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Circular switched monopole arrays for beam steering wireless communications

A. Sibille, C. Roblin and G. Poncelet

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Future mobile communication services which will operate at microwave/millimetre wavelengths require advanced antenna design. The authors here propose circular arrays of monopoles, for beam steering based on electronic switching. They provide appreciable directivity gain, and may allow implementation of angular diversity in communications systems. Satisfactory radiation patterns are computed and measured.

Introduction: The increasing demand for mobile communications services has generated a great deal of research activity aimed at developing new technologies to cope with needs in terms of data throughput, degree of mobility, network access capacity, cost etc. Among the recently expanding areas of research are those concerned with exploiting the enormous spectral resources of the upper microwave and millimetre-wave bands. The European project into mobile broadband services for instance, investigated the feasibility up to 155Mbit/s of mobile communications at carrier frequencies slightly over 60GHz. Less ambitious projects aim at developing wireless LANs at 5.2 or 17.5GHz (HIPERLAN [1]). On account of the spectrum scarcity in lower bands, it is indeed attractive for telecommunication operators to focus on these currently almost unused radio bands. Such communications are generally confined to small cell radius or indoor picocells, for attenuation or because they are inherently local. For that reason, they allow a high degree of frequency re-use and ample service capacity for connected users. However the difficulty of achieving satisfactorily performing systems at these frequencies is extreme, especially as low-cost electronic circuits solutions are required. Poor amplifier power added efficiency or an excessive receiver noise figure are examples requiring expensive solid-state technologies. Furthermore, antennas in the receiving mode suffer from the severe intrinsic limitation of effective aperture, which scales as $\lambda^2$. Therefore in the absence of directivity gain, only reduced signal power can be picked up as the frequency is raised. Secondly, multipath propagation severely limits the bit rate in an indoor/urban environment, due to multiple received echoes and intersymbol interference. Angular diversity obtained by selecting the angle of arrival is a well known way of appreciably minimising this problem.

![Schematic diagram of array antenna and computed radiation pattern](image)

a Schematic diagram array of antenna involving ‘active’ (i.e. connected to source/receiver) or ‘passive’ (i.e. connected to pure reactive load)

b Computed radiation pattern

Thin/thick monopoles only serve to represent passive/active ones, all monopoles being identical in size

● experimentally measured pattern in azimuth