OUTPUT-QUEUE CONTENTS IN A 'MINISLOT' SYNCHRONIZED ATM SWITCH

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SUMMARY
An $N \times N$ switching element with output queuing, as used in a large ATM switching network, is considered. All the inlets of the switching element are synchronized on 'minislots', where a minislot is the fixed-length time unit for the transmission of one 'minicell'. When entering the switching network, an ATM cell is converted into a 'minicell-train', consisting of a fixed number of minicells. Using an active/silent model, it is assumed that on each inlet of the switching element, the number of minicell-trains in an active period and the length of a silent period are both geometrically distributed, and the arriving minicell-trains are uniformly distributed among all the outlets. The performance of the switching element can be obtained by analysing one single output queue, which is modeled as a discrete-time single-server queueing system with train arrivals. In this paper, an upper bound and an approximate expression for the mean queue length are derived. More importantly, an analytical method is developed to obtain a tight upper bound and a good approximation for the tail distribution of the queue length. This analytical method is very useful in buffer dimensioning of ATM switches.

KEY WORDS: ATM switch; discrete-time queue; correlated arrivals; mean queue length; tail distribution; generating functions

1. INTRODUCTION
The asynchronous transfer mode (ATM) is widely considered as a target solution for broadband ISDN which promises to provide a great variety of services from voice, through data to video. In this paper, we consider an $N \times N$ switching element with output queuing,¹ used as a building block in a large ATM switching network.²³ One of the major characteristics of this ATM switching network is as follows. Externally, the switching network is synchronized on slots, where a slot is the fixed-length time unit required to transmit exactly one ATM cell, whereas internally, all the switching elements of the switching network operate in terms of a smaller time unit, called a 'minislot' here, which is the fixed-length time unit for the transmission of one 'minicell'. At the edge of the switching network, each incoming ATM cell is converted into a fixed number, say $m$, of minicells which are transferred through the switching network consecutively. Figure 1 shows an example whereby an ATM cell is internally converted into eight minicells, where the first minicell is the 'self-routing tag (SRT)' which contains all the necessary routing information.¹ The advantages of this internal transfer mode are: (1) to reduce the required buffer capacity for the switching element and the total network cell delay; (2) to facilitate the adaptation to different external cell (or packet) sizes; (3) to optimize the internal characteristics for optimum usage of technologies, which is probably the most important advantage.²

Since all the $N$ inlets of the switching element are synchronized on minislots, the converted cells may arrive on minislot boundaries (see Figure 2). As each converted cell consists of $m$ minicells which arrive at the switching element consecutively, we refer to this minicell arrival process as minicell-train (or train) arrivals in the paper. It is assumed that on each inlet, after the previous train or an empty minislot, a new train (i.e. converted cell) can arrive with probability $q$, and no new train starts with probability $1-q$. In other words, if we model the arriving minicell stream on an inlet by a pattern with two states called the 'active' state and the 'silent' state, which alternate in time, then the number of trains in an active period and the length of a silent period are both geometrically distributed

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with mean values \(1/(1-q)\) and \(1/q\), respectively, i.e.

\[
\text{Prob[length of an active period} = mk \text{ minislots]}
= (1-q)^{k-1} \quad k \geq 1
\]

\[
\text{Prob[length of a silent period} = k \text{ minislots]}
= q(1-q)^{k-1} \quad k \geq 1
\]

The train arrival processes on different inlets are assumed to be statistically independent and have the same traffic characteristics. It is further assumed that each arriving train is destined to one of the \(N\) outlets of the switching element independently and with equal probability \(1/N\). Once the leading minicell of a train is routed to a certain outlet, the remaining \((m-1)\) minicells of this train will also be routed to the same outlet consecutively in the next \((m-1)\) minislots.

The switching element operates on a minislot basis. At each minislot boundary, all the arriving minicells are first routed to their corresponding output queues, and then each output queue transmits one minicell to its outlet, provided that the queue is not empty. Minicells are sent out from the queue according to the rule of first-come–first-served (FCFS) for minicell-trains. Minicells belonging to different trains are not allowed to interleave. The switching element is also assumed to have infinite buffer capacity. It is clear that the \(N\) output queues of the switching element shown in Figure 2 have the same statistical characteristics. So, to obtain the performance of the whole switching element, we only need to analyse one (tagged) output queue, which can be modelled as a discrete-time single-server queueing system with minicell-train arrivals. Here, the performance of the tagged output queue is measured at every minislot boundary just after new minicell arrivals and before a possible minicell departure from the queue.

Two special cases of the above train arrival model were studied before. When \(m = 1\), i.e. there is no cell conversion and the switching element operates on a slot basis, the train arrival process on an inlet of the switching element reduces to the simple Bernoulli arrival process, which was considered in Reference 4. In the case that \(N\) is infinite, the arrival of leading minicells to the tagged output queue becomes independent from minislot to minislot, which has been thoroughly analysed in References 5 and 6. In the paper, we shall consider the case in which \(N\) is finite and \(m > 1\). The purpose of analysing the train arrival model is twofold. First of all, this is the arrival process appearing inside the large ATM switching network. Secondly, the train arrival model is also a kind of bursty traffic model if viewing minislots as slots and minicells as cells, which is different from the commonly used active/silent (or on/off) source model in the distribution of the active period. For this bursty traffic model, the length of an active period can only be multiples of \(m\) slots, whereas in the active/silent source model, the length of an active period in slots is geometrically distributed.

As one will see, the performance analysis of switching elements with the above train arrivals involves solving a \((2m-1)\)-dimensional Markov chain problem. The matrix-geometric solution technique,\(^7\) which was successfully used in many areas, is not very suitable for this case owing to the huge state space requirement \((=N+1)^{2m-2}S\), where \(S\) is the queue length to be reached. The fluid-flow approximation has been widely used in performance evaluation of ATM statistical multiplexers with active/silent traffic sources.\(^8\) However, it is a bit difficult to apply the fluid-flow method in analysing switching elements with the above train arrivals, because, (1) the active period in the train arrival model is not geometrically distributed, and (2) there is routing in switching elements. Therefore, in this paper we try to develop a new analytical method, based on the generating-functions approach, to analyse the performance of the switching element. This method leads to tight upper bounds and good approximations for the mean queue length as well as the tail distribution of the queue length. As the analytical method is easy to use and costs virtually no CPU time, it is very useful in practice.

The rest of the paper is organized as follows. The analytical model of a tagged output queue is
described in Section 2, and a functional equation for the joint probability generating function (p.g.f.) of the state vector is established. In Section 3, the unconditional and conditional (given that the output queue is empty) minicell arrival processes to the tagged output queue in the steady state are discussed. An upper bound and an approximate expression for the mean queue length are derived in Section 4. An analytical method is presented in Section 5 to obtain an upper bound and an approximation for the tail distribution of the queue length. The upper-bound tail distribution and some simulation results are also compared there.

2. ANALYTICAL MODEL AND SYSTEM EQUATIONS

Consider a switching element with minicell train arrivals as shown in Figure 2. Let \( a_{k,n} (b_{k,n}) \) be the random variable representing the number of \( k \)th minicell arrivals (of a train) to the tagged (non-tagged) output queues during minislot \( n \), for \( 1 \leq k \leq m \). Owing to the deterministic relationships between minicells of a train, it is clear that

\[
a_{k,n+1} = a_{k-1,n}, \quad 2 \leq k \leq m
\]

and

\[
b_{k,n+1} = b_{k-1,n}, \quad 2 \leq k \leq m
\]

Define \( \{c_j, j \geq 1\} \) and \( \{d_j, j \geq 1\} \) as two independent sets of i.i.d. Bernoulli random variables, with p.g.f.s

\[
C(z) = 1 - q + qz
\]

and

\[
D(z) = 1 - \frac{1}{N} + \frac{1}{N} z
\]

respectively. From the minicell-train arrival process described above, we have

\[
a_{1,n+1} = \sum_{j=1}^{c_{n+1}} d_j
\]

and

\[
b_{1,n+1} = e_{n+1} - a_{1,n+1}
\]

where

\[
e_{n+1} = \sum_{j=1}^{e_{n+1}} c_j
\]

and

\[
I_{n+1} = \sum_{k=1}^{m-1} a_{k,n} + \sum_{k=1}^{m-1} b_{k,n}
\]

Here, \( e_{n+1} (I_{n+1}) \) denotes the total number of leading (non-leading) minicell arrivals to the switching element during minislot \( n+1 \). In other words, \( I_{n+1} \) denotes the total number of inlets of the switching element occupied during minislot \( n+1 \) with ‘old’ trains, whereas \( e_{n+1} \) indicates the number of inlets occupied with ‘new’ trains during minislot \( n+1 \).

Let \( v_n (n \geq 0) \) be the random variable denoting the number of minicells stored in the tagged output queue at the end of minislot \( n \), just after the possible minicell arrivals during this minislot. The evolution of the tagged output queue is thus governed by the following equation:

\[
v_{n+1} = (v_n - 1)^+ + \sum_{k=1}^{m} a_{k,n+1}
\]

Since \( a_{1,n+1} \) is determined by \( e_{n+1} \), which is given by \( a_{k,n} \) and \( b_{k,n} \) (\( 1 \leq k \leq m - 1 \)) (see equations (7) and (8)), from the above equation together with equations (1) and (2), one can see that \( v_{n+1} \) depends not only on \( v_n \) and the \( a_{k,n} \)s, but also on the \( b_{k,n} \)s.

This means that the states of the inlets sending minicells to the non-tagged output queues still have an impact on the tagged output queue. It is clear that the tagged output queue can be completely characterized by a \( (2m-1) \)-dimensional Markov chain \( (a_{1,n}, \ldots, a_{m-1,n}, b_{1,n}, \ldots, b_{m-1,n}, v_n) \). For convenience, we still use a \( (2m+1) \)-dimensional Markov chain \( (a_{1,n}, \ldots, a_{m,n}, b_{1,n}, \ldots, b_{m,n}, v_n) \) in the following analysis.

Let us define the \( (2m+1) \)-dimensional joint p.g.f. of the random variables \( a_{1,n}, \ldots, a_{m,n}, b_{1,n}, \ldots, b_{m,n}, v_n \):

\[
P_n(x, y, z) \triangleq E[x^{a_1,n} \cdots x^{a_m,n} y^{b_1,n} \cdots y^{b_m,n} z^{v_n}]
\]

where \( x \triangleq (x_1, \ldots, x_m) \), \( y \triangleq (y_1, \ldots, y_m) \) and \( E[.] \) stands for the expectation over the joint distribution of the random variables within the brackets. With this definition, \( P_{n+1}(x, y, z) \) can be obtained by using equations (1), (2), and (9) as

\[
P_{n+1}(x, y, z) = E[(x_1 z)^{a_{1,n+1}}(x_2 z)^{a_{2,n+1}} \cdots (x_m z)^{a_{m-1,n+1}}

y^{b_{1,n+1}} \cdots y^{b_{m,n+1}} z^{v_n - 1}]
\]

Based on equations (5)-(8) and averaging over the \( c_j \)s and \( d_j \)s defined in equations (3) and (4), it follows that

\[
P_{n+1}(x, y, z) = E[(x_1 z/y_1)^{c_{n+1}}(x_2 z/y_2)^{c_{n+1}} \cdots (x_m z/y_m)^{c_{n+1}}

y^{b_{1,n+1}} \cdots y^{b_{m,n+1}} z^{v_n - 1}]
\]

\[
= [C(w)]^{N} E\left[\frac{x_1 z}{C(w)} \cdots \frac{x_m z}{C(w)} \frac{y^{b_{m-1,n}} z^{v_n - 1}}{C(w)}\right]
\]

\[
= \left(\frac{y_2}{C(w)}\right)^{b_{1,n}} \cdots \left(\frac{y_m}{C(w)}\right)^{b_{m-1,n}} z^{e_{n+1} - 1}
\]
where
\[ w \overset{\Delta}{=} y_1 D(x_1/z/y_1) \] (11)

The right-hand side of the above equation can be further expressed in terms of the \( P_n \) function. When \( n \) approaches infinity, we assume that the queuing system can reach a steady state, i.e.
\[ \lim_{n \to \infty} P_{n+1}(x,y,z) = \lim_{n \to \infty} P_n(x,y,z) = P(x,y,z) \]

As \( v_n = 0 \) only implies that \( a_{k,n} = 0 \) but not \( b_{k,n} = 0 \), it is readily shown from the above equation that the steady-state joint p.g.f. \( P(x,y,z) \) satisfies
\[
zP(x,y,z) = [C(w)]^z \left\{ P \left( \frac{x_1 z}{C(w)}, \ldots, \frac{x_m z}{C(w)}, 1, \frac{y_1}{C(w)}, \ldots, \frac{y_m}{C(w)}, 1, z \right) \right. \\
\left. + (z-1)p_0 U \left( \frac{y_1}{C(w)}, \ldots, \frac{y_m}{C(w)}, 1 \right) \right\} \] (12)

where \( p_0 \) is the probability that the tagged output queue is empty and the \( U \) function is defined as
\[ U(y) \overset{\Delta}{=} \lim_{n \to \infty} \sum_{j_1=0}^{N} \ldots \sum_{j_m=0}^{N} \text{Prob}[b_{1,n} = j_1, \ldots, b_{m,n} = j_m | v_n = 0] y_1^{j_1} \ldots y_m^{j_m} \] (13)

In equation (12), the \( P \) function can be reduced to \( (2m-1) \) dimensions by setting \( x_m = 1 \) and \( y_m = 1 \) on both sides of the equation, as we expected. Since the relation between \( b_{k,n} \) and \( v_n \) is indirect, it is very difficult to get an exact expression for \( U(y) \).

In order to get more information about the behaviour of the tagged output queue, as in Reference 9, let the arguments of the \( P \) function on the left-hand side and the right-hand side of equation (12) be chosen equal to each other, i.e. \( x_k = x_{k+1}/C(w), y_k = y_{k+1}/C(w), \) for \( 1 \leq k \leq m-1, \) and \( x_m = 1, y_m = 1, \) where \( w \) is defined in equation (11). As \( x_m = 1 \) and \( y_m = 1, \) it is easily seen that this is equivalent to
\[ \begin{align*}
x_k &= \left( \frac{z}{C(w)} \right)^{m-k} \quad \text{and} \quad y_k = \left( \frac{1}{C(w)} \right)^{m-k} \\
x_k &= \left( \frac{z}{C(w)} \right)^{m-k} \quad \text{and} \quad y_k = \left[ 1/C(y_1 D(z^m)) \right]^{m-k} \\
1 \leq k \leq m \end{align*} \] (14)

From equations (14), \( x_k \) and \( y_k \) can be solved in terms of \( z \). It turns out that there are \( m \) sets of solutions. Here, we only select a set of solutions with the property that \( x_k = 1 \) and \( y_k = 1 \) for \( z = 1, \) which is denoted by \( x_k(z) \) and \( y_k(z), \) respectively. Note that \( x_m(z) = 1 \) and \( y_m(z) = 1. \) Let \( \chi(z) = (\chi_1(z), \ldots, \chi_m(z)) \) and \( \xi(z) = (\xi_1(z), \ldots, \xi_m(z)) \),

choosing \( x = \chi(z) \) and \( y = \xi(z) \) in equation (12) then yields the following normalized result:
\[
P(\chi(z), \xi(z), z) = \frac{(z-1)[1-G'(1)]\varphi(z)G(z)}{z - G(z)} \] (15)

where
\[ G(z) \overset{\Delta}{=} \left[ C(\xi_1(D(z^m))) \right]^N \]
\[
= \left[ 1-q + q\xi_1(1 - \frac{1}{N} + \frac{1}{N} z^m) \right]^N \] (16)

and
\[ \varphi(z) \overset{\Delta}{=} U(\xi(z)) = U(\xi_1(z), \ldots, \xi_m(z)) \] (17)

In equation (15), \( \varphi(z) \) is unknown. In the following, we shall find an approximation as well as an upper bound for \( \varphi(z), \) and then present an analytical approach to obtain upper bounds as well as approximations for the mean and the tail distribution of the queue length, starting from equation (15).

3. UNCONDITIONAL AND CONDITIONAL ARRIVAL PROCESS IN STEADY STATE

Let \( v \) denote the number of minicells stored in the tagged output queue at the end of a minislot in the steady state, and let \( a_k (b_k) \) \( (1 \leq k \leq m) \) denote the number of \( k \)th minicell arrivals (of a train) to the tagged (non-tagged) output queues during this minislot. Let \( p \) be the average traffic load on an inlet of the switching element. From the description of the train arrival model in Section 1, the mean lengths of an active period and a silent period are \( m/(1-q) \) and \( 1/q, \) respectively. So we have
\[
p = \frac{m(1-q)}{[m/(1-q)] + [1/q]} = \frac{mq}{1 + (m-1)q} \] (18)

It is obvious that during an arbitrary minislot in the steady state, a minicell arrives on an inlet with probability \( p \) and no minicell arrives with probability \( 1-p. \) An arriving minicell is the \( k \)th minicell of a train with probability \( 1/m. \) Since all the inlets of the switching element are independent and statistically equivalent, and the arriving trains are uniformly distributed among the \( N \) outlets, it is easy to see that the joint p.g.f. of the \( a_k \)s and the \( b_k \)s can be expressed as
\[ J(x,y) = \left( 1-p + \frac{p}{mN} \sum_{k=1}^{m} x_k + \frac{p}{m} \left( \frac{1}{N} \sum_{k=1}^{m} y_k \right) \right) \] (19)

The marginal p.g.f. \( A_k(x_k) \) of \( a_k \) can be derived
from the above equation by setting \( x_n = 1 \) for \( n \neq k \) and \( y_n = 1 \) (\( 1 \leq n \leq m \)). The marginal p.g.f. \( B_k(y_k) \) of \( b_k \) can be obtained similarly.

Next, consider the conditional marginal probabilities \( \operatorname{Prob}[b_1 = j_1, \ldots, b_m = j_m | \nu = 0] \), which are related to \( \varphi_{\nu}(z) \) defined in equations (17) and (13). Since \( \nu = 0 \) also implies that \( a_k = 0 \), \( 1 \leq k \leq m \), it is intuitive to think that the arrival process to the non-tagged output queues observed when \( \nu = 0 \) is very close to that observed when \( a_k = 0 \), i.e.

\[
\operatorname{Prob}[b_1 = j_1, \ldots, b_m = j_m | \nu = 0] \\
\approx \operatorname{Prob}[b_1 = j_1, \ldots, b_m = j_m | a_k = 0, 1 \leq k \leq m] \quad (20)
\]

This approximation is confirmed by simulations. When \( m = 1 \), the arrival process at the switching element is independent from slot to slot (with a binomial distribution), so the two conditional marginal probabilities in equation (20) actually become equal. Furthermore, in comparison to the unconditional case, it is clear that, under the condition that \( \nu = 0 \), the probability of having more minicells routed to the non-tagged output queues is larger, i.e. for large \( j_k \)s (\( 1 \leq k \leq m \)), we have

\[
\operatorname{Prob}[b_1 = j_1, \ldots, b_m = j_m | \nu = 0] > \operatorname{Prob}[b_1 = j_1, \ldots, b_m = j_m]
\]

Using equation (20), \( \varphi_{\nu}(z) \) defined in equation (17) can be approximately expressed as

\[
\varphi_{\nu}(z) \approx \varphi(z) = \frac{J(0, \ldots, 0, \xi(z), \ldots, \xi_m(z))}{J(0, \ldots, 0, 1, \ldots, 1)} = \left[ 1 - p + \frac{p}{m} \left( 1 - \frac{1}{N} \right) \sum_{k=1}^{m} \xi_k(z) \right]^N
\]

As \( \xi_m(z) = 1 \) and \( \xi_k(z) < 1 (1 \leq k \leq m - 1) \) when \( z \) is real and larger than one (see Appendix 1), for \( z > 1 \), equation (21) leads to

\[
\varphi_{\nu}(z) \approx \varphi(z) = \left[ 1 - p \left( 1 - \frac{1}{N} \right) + \frac{p}{m} \left( 1 - \frac{1}{N} \right) \sum_{k=1}^{m} \xi_k(z) \right]^N
\]

The analysis below is carried out based on equations (20)–(23).

4. APPROXIMATION AND UPPER BOUND FOR THE MEAN QUEUE LENGTH

From the definition of \( P(x,y,z) \) in equation (10) (when \( n \to \infty \)), the p.g.f. \( V(z) \) of \( \nu \) can be expressed as \( V(z) = P(1,1,z) \). To obtain the mean queue length, let us take the first derivative of equation (15) and set \( z = 1 \); after some algebra, we then have

\[
\frac{dP}{dz}
\]

\[
|_{z=1} = \varphi'(1) + G'(1) + \frac{G'(1)}{2[1 - G'(1)]} \quad (24)
\]

where

\[
\frac{dP}{dz} = \frac{\partial P}{\partial x_1} \frac{dX_1}{dz} + \ldots + \frac{\partial P}{\partial x_m} \frac{dX_m}{dz} + \frac{\partial P}{\partial \xi_1} \frac{d\xi_1}{dz} + \ldots + \frac{\partial P}{\partial \xi_m} \frac{d\xi_m}{dz} + \frac{\partial P}{\partial z} \frac{dz}{dz}
\]

Owing to the property of \( \chi_k(z) \) and \( \xi_k(z) \) at \( z = 1 \), for \( 1 \leq k \leq m \), we have

\[
\left. \frac{\partial P}{\partial z} \right|_{z=1} = V'(1) = \bar{\nu}
\]

where \( \bar{\nu} \) is the mean queue length,

\[
\left. \frac{\partial P}{\partial \chi_k} \right|_{z=1} = A_k'(1)
\]

and

\[
\left. \frac{\partial P}{\partial \xi_k} \right|_{z=1} = B_k'(1)
\]

So, equation (24) becomes

\[
\bar{\nu} = \varphi'(1) + G'(1) + \frac{G'(1)}{2[1 - G'(1)]} - \sum_{k=1}^{m-1} \left[ A_k'(1) \chi_k'(1) + B_k'(1) \xi_k'(1) \right]
\]

Here, \( \chi_k'(1) \) and \( \xi_k'(1) \) can be derived from equation (14). \( G'(1) \) and \( G''(1) \), the first and the second derivatives of \( G(z) \) at \( z = 1 \), can be obtained from equation (16). \( A_k'(1) \) and \( B_k'(1) \) can be easily obtained from equation (19). The results are summarized below (\( 1 \leq k \leq m \)):

\[
\xi_k'(1) = -\frac{m(m-k)q}{N[1 + (m-1)q]}
\]

\[
\chi_k'(1) = (m-k) + \xi_k'(1)
\]

\[
A_k'(1) = \frac{p}{m}, \quad B_k'(1) = (N-1)\frac{p}{m}
\]

\[
G'(1) = Nq[\xi_1'(1) + (m/N)] = p
\]

and

\[
G''(1) = \frac{N-1}{N} p^2 + Nq\left[ \xi_1''(1) + \frac{2m}{N} \xi_1'(1) + \frac{m(m-1)}{N} \right]
\]

\[
= \left( 1 - \frac{1}{N} \right) p^2 + (m-1)p \left[ 1 - \frac{(2-p)p}{N} \right]
\]

where
\[ \xi_i^*(1) = (m-1)\xi_i(1) + \left\{ \frac{\xi_i(1)^2}{(m-1)[1+(m-1)q]} \right\} \]

In deriving \( G'(1) \) and \( G''(1) \), we have used the relationship between the traffic load \( p \) and the parameter \( q \) given in equation (18). The only unknown term left in equation (25) is \( \varphi(1) \), where \( \varphi(z) \) is defined in equation (17). If we use equation (22), then the mean queue length can be approximated as

\[ \bar{v} \approx p + \frac{N-1}{N} \left[ \frac{mp^2}{2(1-p)} - \frac{(m-1)p^3}{2(N-p)} \right] \] (26)

Next, let us derive an upper bound for \( \bar{v} \). From the definition of \( U(y) \) in equation (13), \( \frac{\partial U}{\partial \xi_k} \) is the average number of \( k \)th minislot arrivals to the non-tagged output queue during a minislot when \( v=0 \). From equation (21), it is obvious that \( \frac{\partial U}{\partial \xi_k} \) is larger than \( b_k(1) \), the mean value of \( b_k \). Since \( \xi_k(1) < 0 \) for \( 1 \leq k \leq m-1 \) (see Appendix I), we have

\[ \varphi(1) = \sum_{k=1}^{m-1} \frac{\partial U}{\partial \xi_k} \xi_k(1) < \sum_{k=1}^{m-1} b_k(1) \xi_k(1) = \varphi'(1) \]

Using this inequality in equation (25), an upper bound for the mean queue length can be obtained as

\[ \bar{v} < p + \frac{N-1}{N} \frac{mp^2}{2(1-p)} \] (27)

In Table I, we compare the approximate mean queue lengths given by equation (26) with those obtained from simulations, for \( N=4 \), \( m=4 \) and \( 8 \), and various values of the traffic load \( p \). One can see that the two results are very close, which seems to confirm that equation (20) really gives a very good approximation. Figure 3 shows some plots of the mean queue length versus \( p \) for \( N=4 \). It illustrates that, for a given load \( p \), the mean queue length will get larger as \( m \), the number of miniscells in a train, increases. The difference between the approximation and the upper bound will also become larger as \( m \) increases. In the extreme case that \( m=1 \) or \( N=\infty \), equations (26) and (27) lead to the same result, which is the exact mean queue length for these cases and can be found in References 4 and 6, respectively.

5. TAIL DISTRIBUTION OF THE QUEUE LENGTH

Although, in principle, all the information about the behaviour of the tagged output queue is embedded in equation (12), it appears to be very difficult to obtain the exact queue length distribution from this equation. Since for a wide range of queuing systems, including \( G/G/c \), the tail distribution of the queue length has a geometric form, as mentioned in Reference 10, this, of course, is also valid for the tagged output queue, i.e. when the queue length is sufficiently large (e.g. exceeds some threshold \( B \)),

\[ \text{Prob}[\bar{v} = l] = K \gamma^l, \quad l > B \] (28)

In the remainder, we shall present an analytical method to obtain the geometric decay rate \( \gamma \) and an upper bound as well as an approximation for the coefficient \( K \). In this way, the upper-bound and the approximate tail distributions are obtained.

5.1. Geometric decay rate \( \gamma \)

Since \( N \) is finite, from the definition of \( P(x,y,z) \) in equation (10) (when \( n=\infty \)), it seems acceptable that for any finite values of \( x_k \) and \( y_k \) (\( 1 \leq k \leq m \)), \( P(x,y,z) \) has the same poles as \( V(z) (=P(1,1,z)) \),

![Figure 3. Mean queue length versus load p, for N = 4, and m = 1, 4, 8 and 32](image)

<p>| Table I. Mean queue length: upper bound, approximation and simulation, for ( N = 4 ), ( m = 4 ) and ( 8 ) |
|-----------------|--------|-------------|-------------------|</p>
<table>
<thead>
<tr>
<th>Load ( p )</th>
<th>Upper bound</th>
<th>Approximation</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.2750</td>
<td>0.2726</td>
<td>0.2726</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8000</td>
<td>0.7800</td>
<td>0.7826</td>
</tr>
<tr>
<td>0.6</td>
<td>1.9500</td>
<td>1.8785</td>
<td>1.8797</td>
</tr>
<tr>
<td>0.8</td>
<td>5.6000</td>
<td>5.4200</td>
<td>5.4310</td>
</tr>
<tr>
<td>0.9</td>
<td>13.0500</td>
<td>12.7854</td>
<td>12.8380</td>
</tr>
<tr>
<td>( m = 8 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.3500</td>
<td>0.3445</td>
<td>0.3441</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2000</td>
<td>1.1533</td>
<td>1.1536</td>
</tr>
<tr>
<td>0.6</td>
<td>3.3000</td>
<td>3.1332</td>
<td>3.1303</td>
</tr>
<tr>
<td>0.8</td>
<td>10.4000</td>
<td>9.9800</td>
<td>10.0050</td>
</tr>
<tr>
<td>0.9</td>
<td>25.2000</td>
<td>24.5827</td>
<td>24.6690</td>
</tr>
</tbody>
</table>
the p.g.f. of \( v \). As \( \chi_k(z) \) and \( \xi_k(z) \) are finite for any finite value of \( z \) (see equation (14)), it is reasonable that \( V(z) \) and \( P(\chi(z), \xi(z), z) \) in equation (15) might also have the same poles, although by choosing \( x = \chi(z) \) and \( y = \xi(z) \), it is possible that some poles of \( V(z) \) may vanish from \( P(\chi(z), \xi(z), z) \). But we can verify via simulation results that at least \( V(z) \) and \( P(\chi(z), \xi(z), z) \) have the same smallest pole outside the unit circle, which is the most important pole. As stated in References 11 and 12, the tail distribution is dominated by the term of the pole of \( V(z) \) having the smallest modulus outside the unit circle. We denote this dominating pole by \( z_0 \). It is certain that \( z_0 \) must necessarily be real and positive to ensure that the tail distribution is non-negative anywhere. As \( z_0 \) is also the smallest pole of \( P(\chi(z), \xi(z), z) \), similar to Reference 13, we have proved in Appendix II that the real pole \( z_0 \) has multiplicity one. Therefore, with respect to the asymptotic behaviour of the tagged output queue, \( V(z) \) can be approximated as

\[
V(z) \approx \frac{\beta}{z - z_0} \tag{29}
\]

where \( \beta \) is the residue of \( V(z) \) at \( z = z_0 \). Taking the inverse \( z \)-transform of equation (29) then yields

\[
\text{Prob}[v = l] = -\frac{\beta}{z_0} \left( \frac{1}{z_0} \right)^l, \quad l > B \tag{30}
\]

Comparing this to equation (28), we have

\[
\gamma = \frac{1}{z_0} \tag{31}
\]

As \( z_0 > 1 \) is also a pole of \( P(\chi(z), \xi(z), z) \), i.e. a real root of \( z - G(z) = 0 \), where \( G(z) \) is given in equation (16), \( z_0 \) can be calculated numerically. In the real positive domain \( (z > 0) \) it can be proved (see Appendix I) that \( \xi_i(z) > 0 \). So, from \( z - G(z) = 0 \), we have

\[
\xi_i(z_0) = \frac{z_0^{1/N} - (1-q)}{q \left( 1 - \frac{1}{N} \right) \left( 1 + \frac{z_0^m}{N} \right)} \tag{32}
\]

Based on equation (14) and \( G(z_0) = z_0 \), it gives that

\[
z_0 = \left[ \xi_i(z_0) \right]^{1/(m-1)} \left( z_0^{(N+1)/N} \right) \tag{33}
\]

Hence, the smallest pole \( z_0 \) can be obtained by using a repeated substitution algorithm between equations (32) and (33). The decay rate \( \gamma \) is simply given by \( 1/z_0 \).

5.2. Upper bound and approximation for the coefficient \( K \)

If the queue length of the tagged output queue is sufficiently large \((N)\) in a given minislot, it is reasonable to think that the number of minicell arrivals in that minislot \((N)\) has nearly no influence on the total queue length. That is, when the queue length \( l \rightarrow \infty \), we can assume that the state of the queuing system is independent of the arrival process. Let us define the limiting arrival probability function \( \omega(i,j) \) as

\[
\omega(i,j) \triangleq \lim_{l \to \infty} \text{Prob}[a_1 = i_1, \ldots, a_m = i_m, b_1 = j_1, \ldots, b_m = j_m | v = l] \tag{34}
\]

where \( i = (i_1, \ldots, i_m) \) and \( j = (j_1, \ldots, j_m) \). Thus, when \( l \) is very large (e.g. \( l > B \)), we may approximate \( \text{Prob}[a_1 = i_1, \ldots, a_m = i_m, b_1 = j_1, \ldots, b_m = j_m | v = l] \) by \( \omega(i,j) \).

Let \( p(i,j,l) \) denote \( \text{Prob}[a_1 = i_1, \ldots, a_m = i_m, b_1 = j_1, \ldots, b_m = j_m, v = l] \), using equation (34), \( P(x,y,z) \) defined in equation (10) \((N)\) can be approximately expressed as

\[
P(x,y,z) \approx \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{l=0}^{B} \sum_{m=0}^{B} \text{Prob}[v = l] \cdot z_0^m \left( x_{i1} y_{j1} \ldots x_{im} y_{jm} \right) + \Omega(x,y) \left( V(z) - \sum_{l=0}^{B} \text{Prob}[v = l] z_0^l \right) \tag{35}
\]

where \( \Omega(x,y) \) is the p.g.f. for \( \omega(i,j) \). When \( x = \chi(z) \) and \( y = \xi(z) \), we know from the previous subsection that \( P(\chi(z), \xi(z), z) \) and \( V(z) \) have the same smallest pole \( z_0 \). The threshold value \( B \) in the above equation can be set to be extremely large. Since \( \chi_k(z) \) and \( \xi_k(z) \) are finite \((1 \leq k \leq m)\), as long as \( B < \infty \), the other terms in the above equation are finite. Thus, multiplying by \( (z - z_0) \) on both sides of the above equation and taking the \( z \rightarrow z_0 \) limit, together with equations (28)–(30), one can easily obtain

\[
K = -\frac{\beta}{z_0} = \frac{[1-G'(1)](z_0-1)\psi(z_0)}{[G'(z_0)-1]\Omega(\chi(z_0), \xi(z_0))} \tag{35}
\]

In the above equation, \( G'(1) = p \), \( G'(z_0) \) can be derived from equation (16). An explicit expression for \( \Omega(x,y) \), the joint p.g.f. associated with the limiting arrival process defined in equation (34), is given in Appendix III. So the only unknown value in equation (35) is \( \psi(z_0) \), which is very difficult to obtain owing to a great number of unknown boundary probabilities in equation (13). Since \( \psi(z_0) < \psi(z_0) \) (see equation (23)), if we replace \( \psi(z_0) \) by \( \psi(z_0) \), then we get an upper bound for the coefficient \( K \), denoted by \( K_u \). Similarly, an approximation for \( K \) (denoted by \( K_a \)) can be obtained by using \( \psi(z_0) \equiv \psi(z_0) \).

To conclude this section, the tail distribution of the queue length can be approximated by a geometric form for which an explicit upper bound and an approximation can be obtained, i.e. for sufficiently large \( l \).
\[
\text{Prob}[v=l] \approx K_a \left( \frac{1}{z_0} \right)^{1/\gamma} < K_a \left( \frac{1}{z_0} \right)^{1/\gamma}
\]  

(36)

where \(z_0\) is determined by equations (32) and (33) and \(K_a\) (\(K_u\)) can be calculated from equation (35) by replacing \(\varphi_v(z_0)\) by \(\varphi(z_0)\) (\(\varphi_u(z_0)\)).

5.3. Comparison with simulation results

In practice, we are mainly interested in the probability of the queue length exceeding a proposed buffer size \(S\); from equation (36), we have

\[
\text{Prob}[v>S] = \frac{K_a}{z_0-1} \left( \frac{1}{z_0} \right)^S < \frac{K_a}{z_0-1} \left( \frac{1}{z_0} \right)^S
\]  

(37)

Since \(\varphi_v(z_0)\) and \(\varphi(z_0)\) (and hence, \(K_u\) and \(K_a\)) are very close to each other, we only compare the upper-bound complementary tail distribution of the queue length given by equation (37) with simulation results in Figures 4–6. The simulation results are not stable in the low probability area (roughly starting from \(10^{-5}\)) due to the limited simulation time. The small difference between \(K_u\) and \(K_a\) explains that the upper-bound tail distribution is very tight, as shown in Figures 4–6. Furthermore, the simulation results confirm that the analytical approach used in Subsection 5.1 to obtain \(z_0\), the pole of \(V(z)\) with the smallest modulus outside the unit circle, is correct. It is worth mentioning that in the whole derivation, we always say that equation (28) is valid for sufficiently large queue lengths; we see from Figures 4–6 that the queue-length distribution is already very close to its tail distribution even for small queue lengths. One can also see from Figure 4 that, for a given traffic load, the probability of the queue length exceeding a certain buffer size becomes larger if the correlation in the minicell arrival process gets stronger (i.e. if \(m\), the number of minicells in a train, gets larger). Note that \(m=1\) corresponds to an uncorrelated minicell arrival process.

6. CONCLUSIONS

An \(N \times N\) switching element with output queuing, used in a large ATM switching network, is considered in this paper. Each output queue of the switching element is modelled as a discrete-time single-server queuing system with train arrivals, which has been analysed by means of a generating-functions approach. Since it is very difficult to derive the exact queue-length distribution, we give an upper bound and an approximate expression for the mean queue length. We also present an analytical
method to obtain an upper bound as well as an approximation for the tail distribution of the queue length. The analytical method is validated by the simulation results. It is illustrated that this method can give a very good approximation and a tight upper bound for the tail distribution of the queue length.

Since for most of the discrete-time queuing systems, the tail distribution of the queue length has a geometric form, the analytical method developed in this paper could also be used for other types of correlated arrivals. For instance, the extension to analyse switching elements with more general active/silent traffic models (e.g., correlated train arrivals) is quite straightforward.

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APPENDIX I: PROPERTIES OF $\chi_k(z)$ AND $\xi_k(z)$ FOR $1 \leq k \leq m - 1$

Consider $g(y_1) \equiv y_1[C(y_1 D(zm))^{m-1} - 1$ for real non-negative values of $z$, where $C(.)$ and $D(.)$ are defined in equations (3) and (4), respectively. The first derivative of $g(y_1)$ is given by

$$g'(y_1) = [1-q + qy_1 D(zm)]^{m-2}$$

$$[1-q + m q y_1 D(zm)]$$  \hspace{1cm} (38)

Since $g(0) = -1 < 0$, $g(+\infty) = +\infty$ and $g'(y_1) > 0$ for $y_1 > 0$, $g'(y_1)$ crosses the $y_1$-axis in the positive region once, which means that there always exists exactly one real positive zero of $g(y_1)$ for $z > 0$. It is obvious that this zero is a continuous function of $z$. As $\xi_1(1) = 1$ is a zero of $g(y_1)$ when $z = 1$, $\xi_1(z)$ is the real positive zero of $g(y_1)$, i.e., the only real positive solution for $y_1(z)$ in equation (14). Let $\chi_k(z) = \chi_k(z) (1 \leq k \leq m - 1)$ and $\xi_k(z) = \xi_k(z) (2 \leq k \leq m - 1)$ be the set of solutions of equation (14), which are related to $\xi_1(z)$; we have $\chi_1(z) = 1$ and $\xi_1(z) = 1$. The first derivatives of $\chi_k(z)$ and $\xi_k(z)$ can be derived from equation (14) as follows:

$$\chi'_k(z) = (m-k)z^{m-k-1}\xi_k(z) + z^{m-k}\xi'_k(z)$$

$$\xi'_k(z) = -m(m-k)qz^{m-1}\xi_k(z)\xi_k(z)$$

$$N[1-q + mq\xi_k(z)D(zm)]$$

(39)

(40)

Since $\xi_1(z) > 0$, from equation (14), we have $\xi_k(z) > 0$ and $\chi_k(z) > 0$. So $\xi_k(z) < 0$. As $\xi_1(1) = 1$, $\xi_1(z) < 0$ for $z > 1$. Based on equations (39) and (40), one can also obtain $\chi_k(z) > 0$. Since $\chi_1(1) = 1$, $\chi_k(z) > 1$ for $z > 1$.

APPENDIX II: PROPERTIES OF THE SMALLEST REAL POLE $z_0$ OF $V(z)$

Let $h(z) \equiv G(z) - z = [1-q + qy_1 D(zm)]^{m-1} - z$ for $z \geq 0$, where $y_1$ is implicitly given by equation (14), i.e.

$$y_1[1-q + qy_1 D(zm)]^{m-1} = 1$$

(41)

where $D(.)$ is defined in equation (4). There are $m$ solutions for $y_1$ in terms of $z$ in the above equation ($m$ 'branches') and it is obvious that exactly one of them has the property that $y_1 = 1$ when $z = 1$, which is denoted by $\xi_1(z)$. In the following, we shall prove that the smallest real zero $z_0 (>1)$ of $h(z)$ is completely determined by the branch $y_1 = \xi_1(z)$, and its multiplicity is one.

First, assume that $h(z)$ has real zeros (>1) for some solutions of $y_1$ in equation (41). Let $z^*>1$ be such a zero of $h(z)$ when $y_1 = \xi_1(z)$. Based on equation (41) and $h(z^*) = 0$, one can easily prove that $y_1(z^*)$ has two properties: (i) $y_1(z^*)$ must be real; (ii) $|y_1(z^*)| < 1$. So, either $-1 < y_1(z^*) < 0$ or $0 < y_1(z^*) < 1$. Let us assume that $z^* > 1$ are two real zeros (>1) of $h(z)$ such that $0 < y_1(z^*_1) < 1$, $-1 < y_1(z^*_2) < 0$. From equation (41) and $h(z^*_1) = 0$, one can see that $z^*_1$ exists only when $m$ and $N$ are even. In this case, $1-q + qy_1(z^*_1)D(z^*_1) = 0$. Using equation (41) and $h(z^*_1) = 0$, it can be shown that

$$z_1^{1/N} \left[ qD(z^*_1) \right] = 1 - q$$

(42)

Similarly, for $z_0$, we have

$$z_0^{1/N} \left[ 1 - qD(z_0^m) \right] = 1 - q$$

(43)

Since $D(z^m)/(z^{m/N})$ is an increasing function of $z$, from equations (42) and (43), it is clear that $z^*_1 > z_0$. Hence, if $h(z)$ has real zeros (>1) for which the corresponding $y_1$-values are positive, then the smallest real zero (>1) of $h(z)$ must be one of them. We next prove that there always exists exactly one such zero of $h(z)$, which must therefore necessarily be the smallest pole $z_0$ of $V(z)$.

In Appendix I, we have verified that there always exists exactly one real positive solution of $y_1$ which is less than one for $z > 1$ and this solution is $\xi_1(z)$. When $y_1 = \xi_1(z)$, it is obvious that $z=1$ is a zero of $h(z)$. Since $h'(1) = G'(1) - 1 = p - 1 < 0$, if we can prove that $h(z) > 0$ when $z$ is sufficiently large and $h(z)$ is a concave function, i.e. $h''(z) > 0$, then it is sure that there is another zero of $h(z)$, i.e. $z_0$, which is larger than one and has multiplicity one. Suppose that $h(z) < 0$ for all $z > 1$; this gives that $z^{(m-1)/N} < N/q$, which is contradictory to the fact that $z$ can be any positive value. Thus $h(z) > 0$ when $z$ is sufficiently large. From equation (40), the second derivative of $\xi_1(z)$ can be obtained as
\[ \xi(z) = \left[ \frac{\xi(z)}{\xi(z)} \right]^{2} + \frac{m}{m-1} \frac{1-q+q \xi(z)D(z^{m})}{1-q+q \xi(z)D(z^{m})} + \frac{m-1}{z} \xi(z) \]  \hspace{1cm} (44)

The second derivative of \( h(z) \) is given by

\[ h''(z) = N(N-1)[1-q+q \xi(z)D(z^{m})]^{N-2} q^{2}[\xi(z)D(z^{m})]^{N-1} + \frac{m}{N} \xi(z)z^{m-1} \]  \hspace{1cm} (45)

It is clear that the first term of \( h''(z) \) is positive. Based on equations (40) and (44), it can be shown that the second term of \( h''(z) \) is also positive, so \( h''(z) > 0 \) for \( z > 0 \).

APPENDIX III: THE p.g.f. \( \Omega(x,y) \) OF THE LIMITING ARRIVAL PROCESS

Let \( p(i,j,l) = \text{Prob} \{ a_i = i_1, \ldots, a_m = i_m, b_i = j_1, \ldots, b_m = j_m, v = l \} \) and \( p(i,j) \) be the steady-state conditional arrival probability function when the queue length is \( l \). Using the relationships of one-step transition, for large \( l \) (e.g. > \( N \)), the joint probability can be expressed as

\[ p(i,j) = \sum_{i_1} \sum_{i_2} \cdots \sum_{i_k} \pi_{1,i_1} \cdots \pi_{k,i_k} (i,j) \]  \hspace{1cm} (46)

where \( i^* = (i_1, \ldots, i_m, i_{k}) \), \( j^* = (j_1, \ldots, j_m, j_{k}) \) and \( \pi_{1,i_1} \cdots \pi_{k,i_k} \) is the (non-zero) one-step transition probability of the train arrival process. As \( p(i,j,l) = p(i,j) \text{Prob} \{ v = 1 \} \), taking the limit for \( l \to \infty \) and using equations (28), (31) and (34), we obtain from equation (45) that

\[ z_0 \omega(i,j) = \sum_{i_1} \sum_{i_2} \cdots \sum_{i_k} \pi_{1,i_1} \cdots \pi_{k,i_k} \omega(i^* j^*)z_1 \cdots z_{k-1} \]  \hspace{1cm} (47)

From this equation, it can be shown that the p.g.f. \( \Omega(x,y) \) associated with the limiting arrival probability function \( \omega(i,j) \) satisfies

\[ z_0 \Omega(x,y) = [C(w_0)]^{N} \Omega(z_2 \cdots z_{m-1} \cdots z_{m} = 1, \frac{y_1}{C(w_0)}, \ldots, \frac{y_m}{C(w_0)}, 1] \]  \hspace{1cm} (48)

where \( w_0 = y_1D(x_{i_{k}}y_1) \), and \( C(.) \) and \( D(.) \) are defined in equations (3) and (4), respectively.

In equation (12), setting \( z=1 \), the left-hand side equals the p.g.f. \( J(x,y) \) of the steady-state unconditional arrival process, i.e. \( J(x,y,1) = J(x,y) \), which is given in equation (19). If \( z_0 \) in equation (47) were equal to one, then \( \Omega(x,y) \) would be just equal to \( J(x,y) \). Since \( \Omega(x,y) \) is also a polynomial of degree \( N \), it is clear from the above equation that \( \Omega(x,y) \) has the same form of expression as \( J(x,y) \), i.e.

\[ \Omega(x,y) = \left[ 1 - \sum_{k=1}^{m} (\sigma_{k}^{*}x_{k}^{*} + \mu_{k}^{*}y_{k}^{*}) \right]^{N} \]  \hspace{1cm} (49)

Substituting equation (48) into (47), the parameters \( \sigma_{k}^{*} \) and \( \mu_{k}^{*} \) \( 1 \leq k \leq m \) can be easily solved for as

\[ \sigma_{k}^{*} = \frac{q_{20}}{Nz_{0}^{k+q_{20}} f(z_{0}^{1\frac{k-1}{N}}) + \frac{N-1}{z_{0} f(z_{0}^{1\frac{k}{N}})}} \quad \text{and} \]  \hspace{1cm} (50)

\[ \mu_{k}^{*} = z_{0}^{-\frac{k}{k(k-1)}} \left( \frac{N-1}{z_{0}} \right) \sigma_{k}^{*}, \quad 1 \leq k \leq m \]  \hspace{1cm} (51)

where

\[ f(x) \approx \frac{x^{m-1}}{z_{0}^{l-1}} \]

Comparing equation (47) with (12) tells us that equation (47) can be 'directly' written from equation (12). So, to derive the p.g.f. \( \Omega(x,y) \) of the limiting arrival process is no longer difficult.

REFERENCES

9. H. Bruneel, ‘Queueing behavior of statistical multiplexers


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